Motion planning algorithms for stratified kinematic systems with application to the hexapod robot [5]

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The paper addresses the motion planning problem (MPP) of kinematic systems where the configuration space is stratified.

Roughly speaking, the MPP can be defined as a search among all feasible trajectories of the system that satisfy a desired control objective i.e. to move the system from a known initial state (or configuration) x_i to a desired final state x_f. Feasibility means that, along such trajectories, the system variables identically satisfy the system equations.

In this paper we restrict our attention to kinematic and stratified systems. Typically, such systems arise in the presence of non-holonomic constraints [4]. The state equation of a kinematic system reads:

$$\dot{x} = \sum_{i=1}^{m} g_i(x)u_i \quad x \in \mathcal{R}^n \tag{2}$$

where x is the state variable, u is the input variable and g_i are (sufficiently regular or smooth) vector fields on $\mathcal{R}^n$ ($n \geq m$).

The systems considered here are also said to have stratified configuration space, notion introduced by Goodwine et. al. in [3]. For such systems, the configuration space is divided in strata (set of intersecting submanifolds of the configuration manifold) such that the equations of motions may differ for each strata and change discontinuously. We call the bottom stratum the submanifold with the highest co-dimension.

Legged robotic structures are typical examples for stratified systems where the possible leg contact configurations are defined by constraints. Since these constraints define submanifolds in the configuration space where the equations of motions are different, one obtains a stratified system.

The main problem arising in the control of such systems is that the bottom startum is not controllable. Therefore one has to switch to lower codimension strata to find feasible trajectory between two points of the bottom stratum. The switch changes in a discontinuous manner the equations of motions, situation which is not treated by conventional motion planning algorithms working on smooth configuration spaces. Thus these algorithms must be adapted (extended) to work with stratified systems.

A general motion planning algorithm, proposed by Lafferriere et al. [5] is used in [3] to solve the MPP for stratified systems. This algorithm uses piecewise constant inputs but it is imprecise if the Lie algebra generated by the vector fields g_i fails to be nilpotent. (Recall that in this case the Campbell-Baker-Hausdorff formula can be used to achieve any desired accuracy.) Notice also that the plannified trajectory may be composed from a prohibitively huge number of pieces and thus difficult to be realized in practice. Finally, let us note also that the algorithm does not give precise information about the actual trajectory of the system which can be nevertheless calculated by integrating the state equation with the plannified input sequence.

Motion planning can be easily solved for a restricted class of kinematic systems which are called differentially flat [1, 2]. For such systems, the MPP is reduced to a simple interpolation problem in the space of the flat output where the set of feasible trajectories is unconstrained in the sense that there is a one-to-one correspondence between sufficiently smooth trajectories of the flat output and feasible trajectories of the system.

We adapt this latter method to be used for stratified systems. Thus we construct trajectories made of pieces along which all but one input is zero. This can be done if we connect the initial and final points in the space of the flat output using the images of the flows associated to the control vector fields g_i in this space. Then the motion planning is reduced to a geometric problem, provided that the flows can be easily obtained. (This can be made off-line.) Due to the fact that we have direct influence on the geometry of the trajectory, obstacle avoidance can be also treated explicitly.

The application to the above methods is illustrated and compared using the example of the hexapod (six-legged) robot.

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