Investigation of Point–Based Image Registration Methods Assuming Rigid–Body and Linear Motions

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Image registration is used to match two independently acquired images.

The geometrical transformation is to be found that maps a *floating image* in precise spatial correspondence with a *reference image*. Point–based registration requires the matching of a set of 3D points in the reference image with a homologue set of 3D points in the floating image. We call these points used for registration fiducials. Fiducial points can be obtained from external fiducial markers or from internal anatomical landmarks.

Registration techniques involve searching over the space of transformations of a certain type to find the optimal one–to–one mapping for a particular problem. The major classes of 3D transformations are *rigid–body transformation* when only translations and rotations are allowed, *affine transformation*, which maps parallel lines onto parallel ones, and *nonlinear*, *curved* or *elastic transformation* i.e., which maps straight lines onto curves.

Point–based registration might find imperfect matching due to the presence of error in localizing the fiducials. There are some papers dealing with the analysis of point–based registration. Emphasis is to be put that each of these papers assumes only rigid–body transformation. Maurer et al. [8] proposed three types of measures of error: *Fiducial localization error* (FLE), which is the error in determining the positions of the fiducials, *fiducial registration error* (FRE), which is the root mean square distance between corresponding fiducials after registration, *target registration error* (TRE), which is the distance between corresponding points representing surgical targets after registration. Note that point-based registration methods minimize FRE. But using FRE as measure of registration accuracy is unreliable and may be misleading, thus investigations were focussed on TRE in the last decade [4, 9].

There are two important results concerning rigid–body registration errors:

- **Result 1.** For a fixed number of fiducials, TRE is proportional to FLE [3, 4, 7, 9],
- **Result 2.** TRE is approximately proportional to $1/\sqrt{n}$ with n being the number of fiducials [4, 6, 9].

It is obvious that TRE is sensitive to the location of target points and the location of the fiducials. Maurer et al. [9] used numerical simulations to give qualitative insight into this dependence. In addition, they realized that TRE depends on the fiducial configurations and the distances between fiducials via the examinations of four configurations of four fiducials.

Fitzpatrick et al. [4] gave exact expression for approximating TRE assuming rigid–body registration problem, thus proving **Result 1** and **Result 2**, and answering how does TRE depend on the relative positions of target and fiducial points. They also stated that TRE is inversely proportional to the scale factor used for scaling fiducials.

We investigate the following four point–based registration methods:

- RB1

  The solution of Arun et al. [1] is implemented for determining a rigid–body transformation given by a 3×3 rotation matrix and a translation vector (3×1 matrix). The rotation matrix is computed using the singular value decomposition (SVD) of the covariance matrix of the centroid–subtracted position vectors of the corresponding fiducials. The translation vector is calculated as the difference between the centroids of the two sets of points. This method is used by several authors [5, 9, 11, 14]. It is regarded as the most popular point based registration approach.

- RB2

  Our implementation utilizes the Levenberg–Marquardt technique [10] for finding the 6 parameters of rigid–body transformations. There is a possibility that this iterative minimization might fail because of local minima in the parameter space. This method is used by Zuk et al. [13], too.

- LIN
  The method proposed by Tanács et al. [12] allows more freedom. It is to find affine transformations given by the 12 unknown elements of 4×4 transformation matrices (using homogeneous coordinates).
- TPS
  We implemented the thin–plate spline interpolation proposed by Bookstein [2]. It is capable of finding nonlinear transformations given by $3 \cdot (n + 4)$ parameters with n being the number of fiducials. Note that the complexity of reslicing (i.e., applying the found transformation for the reslice image) does not depend on n in the case of the other three considered methods. It is not true for the thin–plate spline warping: the required time of reslicing is proportional to n.

Numerical simulations are made assuming both rigid–body and affine motions.

We extended the observations according to FLE and TRE for point based registration methods with different search spaces, assuming not only rigid–body but affine motions, as well. Our examinations confirm **Result 1** for all the four methods, and **Result 2** for rigid–body methods. Another result in this work is that TRE depends on the volume spanned by the fiducials:

- **Result 3.** TRE is inversely proportional to the mean distance from the centre of gravity.

We compared two rigid–body methods RB1 and RB2. Since RB2 is an iterative method, its result depends on the initial guess and numerical errors are accumulated. In some cases it may diverge. That is why using the direct method (RB1) is recommended.

If there is a precise rigid–body correspondence between the two sets of points used for registration, then any linear or nonlinear method is able to find the adequate transformation, since a rigid–body transformation is a special kind of linear or nonlinear one. It is not hold in real registration problems due to the presence of FLE. It can be stated that RB1 and RB2 (i.e., the methods restricted to rigid–body transformations) are better than LIN and TPS (i.e., the methods which allow more freedom) if a rigid–body transformation is assumed. Similarly, LIN is better than TPS if a linear motion is assumed. That is because transformations having more degrees of freedom than necessary can incorporate a wider range of fake motions induced by FLE.

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References

[1] Arun, K.S., Huang, T.S., Blostein, S.D.: Least–Squares Fitting of Two 3–D Point Sets. IEEE Trans. on Pattern Analysis and Machine Intelligence **9** (1987) 698–700

[2] Bookstein, F.L.: Principal warps: thin–plate splines and the decomposition of deformations. IEEE Trans. on Pattern Analysis and Machine Intelligence **11** (1989) 567–585

[3] Evans, A.C., Marrett, S., Collins, D.L., Peters, T.M.: Anatomical–functional correlative analysis of the human brain using three dimensional imaging systems. In: Medical Imaging: Image Processing, vol. Proc. SPIE 1092 (1989) 236–246

[4] Fitzpatrick, J.M., West, J.B., Maurer, C.R.: Predicting error in rigid–body point-based registration. IEEE Trans. on Medical Imaging **17** (1998) 694-702

[5] Hill, D.L.G.: Combination of 3D Medical images from Multiple Modalities. Ph.D. Thesis, Image Processing Group Radiological Sciences UMDS, London (1993)

[6] Hill, D.L.G., Hawkes, D.J.: Medical image registration using knowledge of adjacency of anatomical structures. Image and Vision Computing **12** (1994) 173–178

[7] Mandava, V.R., Fitzpatrick, J.M., Maurer, C.R., Maciunas, R.J., Allen, G.S.: Registration of multimodal volume head images via attached markers. In: Medical Imaging: Image Processing, vol. Proc. SPIE 1652 (1992) 271–282

[8] C.R. Maurer, J.J. McCrory, J.M. Fitzpatrick, Estimation of accuracy in localizing externally attached markers in multimodal volume head images. In: Medical Imaging: Image Processing, vol. Proc. SPIE 1898 (1993) 43–54

[9] Maurer, C.R., Fitzpatrick, J.M., Wang, M.Y., Galloway, R.L., Maciunas, R.J., Allen, G.S.: Registration of head volume images using implantable fiducial markers. IEEE Trans. on Medical Imaging **16** (1997) 447–462

[10] Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T.: Numerical Recipes in C: The Art of Scientific Computing. Cambridge University Press, second edition (1992)

[11] Strasters, K.C., Little, J.A., Buurman, J., Hill, D.L.G., Hawkes, D.J.: Anatomical landmark image registration: validation and comparison. In: CVRMed–MRCAS'97, Lecture Notes in Computer Science, Vol. 1205. Springer-Verlag, Berlin Heidelberg New York (1997) 161–170

[12] Tanács, A., Palágyi, K., Kuba, A.: Medical image registration based on interactively defined anatomical landmark points. Int. J. Machine Graphics & Vision **7** (1998) 151–158

[13] Zuk, T.D., Atkins, M.S.: A Comparison of manual and automatic methods for registering scans of the head. IEEE Trans. on Medical Imaging **15** (1996) 732–744

[14] West, J., Fitzpatrick, M., Wang, M.Y., et al.: Comparison and evaluation of retrospective intermodality image registration techniques. In: Medical Imaging: Image Processing, Proc. SPIE 2710 (1996) 332–347