Generalised Subspace based Method for Steady State Analysis of QBD-M Processes

Hung T. Tran and Tien V. Do

The search of numerical solutions for queueing models in the performance analysis of computer systems and communication networks has practical importance and is always a hot research topic. In recent years, the extensive application of two-dimensional queueing systems which state is described by a phase and a level has been witnessed, particularly on the performance evaluation of ATM (Asynchronous Transfer Mode) systems. If the level transitions are only possible between adjacent ones, such queueing systems are called QBD (Quasi Birth-Death) processes. For the steady state analysis of this class of two-dimensional Markov chains several efficient methods have been developed and improved over recent years. For example, one can mention the method of Naoumov et al., the spectral expansion method, the ETAQA method and many others [1, 2, 3, 4].

However, there are only a few of works dealing with such two-dimensional queueing systems in which upper-bounded batches (either arrival or departure) occur, i.e. multiple jumps in level dimension are possible. Such systems (referred as QBD-M processes) can be considered as an extension of QBD processes and therefore after some manipulations (e.g. re-blocking), computation methods developed for QBD processes can be applied. Nevertheless, from the technical point of view direct methods may have some advantages over the former ones and that's why it's worth devoting research efforts in this direction.

Towards to the aim addressed before, this paper proposes a direct computational method for computing steady state distribution of QBD-M processes. This method is based on the theory of generalized invariant subspace which has been first introduced and applied by Nail Akar et al. in [3, 5] to teletraffic problems. Moreover, the capability and applicability of this method will be discussed and demonstrated through a case study and will be compared with other existing computational ones, such as Naomouv's method and spectral expansion applied after re-blocking, the iterative method proposed in [6].

References

[1] Valeri Naoumov, Udo R. Krieger, and Dietmar Warner. Analysis of a Multi-Server Delay-Loss System With a General Markovian Arrival Process. *Matrix-analytic methods in stochastic models*, 1997.

[2] R. Chakka. *Performance and Reliability Modelling of Computing System Using Spectral Expansion*. PhD thesis, University of Newcastle upon Tyne, 1995.

[3] Nail Akar and Khosrow Sohraby. Finite and Infinite QBD Chains: A Simple and Unifying Algorithmic Approach. *in Proceedings of IEEE INFOCOM*, 1997.

[4] Gianfranco Ciardo and Evgenia Smirni. ETAQA: An Efficient Technique for the Analysis of QBD-processes by Aggregation. In *Proceeding of Performance '99*, 1999.

[5] Nail Akar, Nihat Cem Oguz, and Khosrow Sohraby. Matrix-geometric Solution of M/G/1 Type Markov Chains: A Unifying Generalized State-space Approach. *in IEEE JSAC*, 16, 1998.

[6] Hung T. Tran and Tien V. Do. A new iterative method for systems with batch arrivals and batch departures. In *Proceedings of Communication Networks and Distributed Systems Modeling and Simulation, CNDS'00*, San Diego, USA, 2000.