On a Class of Cyclic-Waiting Queueing Systems with Refusals

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Based on a real problem connected with the landing of aeroplanes we investigate a special queueing system, where peculiar conditions prevail. In such systems a request for landing can be serviced upon arrival if the system is free. When other planes are using the runway or waiting to land, the entering plane has to start a circular manoeuvre and can put its further requests when it comes to the starting point of its trajectory. Because of possible fuel shortage it is quite natural to use the *FIFO* rule.

In his works Lakatos has extensively investigated such type of queueing systems, namely where the service of a request can be started upon arrival (in case of a free system) or at times differing from it by multiples of the cycle time T (in case of a busy server). In [1] he considered a system with Poisson-arrivals and uniformly distributed service time. As a generalization, in [2] a special system which serves customers of two different types, was examined. Both types of customers form Poisson processes, and their service time distributions are exponential. In the system only one customer of first type can be present, it can only be accepted for service in the case of a free system, whereas in all other cases the requests of such customers are turned down. There is no such restriction on customers of second type; they are serviced immediately or join a queue in case of a busy server. In this paper we are going to consider the same system but service times are uniformly distributed.

To elaborate the mathematical description of the system we make the following proposals. In the system there might be idle periods, when the service of a request is completed, but the next one has not reached its starting position. We consider these periods part of the service time, making the service process continuous in such way. We also make a restriction on the boundaries of the intervals of the uniform distributions: they are multiples of the cycle-time. This assumption does not violate the generality of the theory, but without it formulae are much more complicated.

For the description of the system we use the *embedded Markov-chain technique*, i. e. we consider the number of customers in the system at moments just before the service of a new customer begins. For this chain we introduce the following transition probabilities:

- a_{ji} – the probability of appearance of i customers of second type at the service of a j-th type customer ($j = 1, 2$) if at the beginning there is only one customer in the system;
- b_i – the probability of appearance of i customers of second type at the service of a second type customer, if at the beginning of service there are at least two customers in the system;
- c_i – the probability of appearance of i customers of second type after free state.

We formulate the results of the paper in the following

Theorem. *Let us consider a queueing system with two types of customers forming Poisson-processes with parameters λ_1 and λ_2, the service times are uniformly distributed in the intervals $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$, respectively ($\alpha_1, \beta_1, \alpha_2, \beta_2$ are multiples of cycle-time T). There is no restriction on customers of second type; however customers of first type may only join the system when it is free (and only one of them can be present at every instant), all other requests of this type are refused. The service of a customer may start at the moment of its arrival (in case of a free system) or at moments differing from it by multiples of cycle time T; and the* FIFO *rule is obeyed. We define an embedded Markov-chain, whose states correspond to the number of customers in the system at moments just before starting a service.*

The matrix of transition probabilities of this chain has the form:

$$\begin{pmatrix} c_0 & c_1 & c_2 & c_3 & \dots \\ a_{20} & a_{21} & a_{22} & a_{23} & \dots \\ 0 & b_0 & b_1 & b_2 & \dots \\ 0 & 0 & b_0 & b_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The condition of existence of ergodic distribution is the fulfilment of inequality:

$$\frac{\lambda_2(\alpha_2 + \beta_2 + T)}{2} < 1$$

The limit distribution while $T \to 0$ is also given.

References

[1] L. Lakatos: On a Cyclic-Waiting Queueing System. Theory of Stochastic Processes Vol. 2 (18) no. 1-2, 1996 pp. 176-180

[2] L. Lakatos: A Special Cyclic-Waiting Queueing System With Refusals. J. Math. Sci. (New York) (to appear)

[3] L. Lakatos: Limit Distributions for Some Cyclic-Waiting Queueing Systems. Theory of Stochastic Processes (to appear)