New Interval Methods for Constrained Global Optimization: Solving 'Circle Packing' Problems in a Reliable Way

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The talk gives the theoretical and numerical results of solving inequality constrained global optimization test problems with interval Branch-and-Bound methods using new techniques.

In [1, 3] a new heuristic decision index was discussed for *unconstrained* problems and investigated in detail. This index has the form of $p\hat{f}(\boldsymbol{X}) := (\hat{f} - \underline{\boldsymbol{f}}(\boldsymbol{X}))/w(\boldsymbol{f}(\boldsymbol{X}))$, where $\boldsymbol{X}$ is an interval vector, $\hat{f}$ is an approximation of the global minimum value and $\boldsymbol{f}$ denotes the interval inclusion function of the objective function. The index measures the relative position of the minimum within the inclusion $\boldsymbol{f}(\boldsymbol{X})$ and it is suitable to be applied as a subinterval selection criterion and as a part of the subdivision rule as a decision factor.

J. F. Hernández proposed the idea of extending this index for constrained problems by taking the effect of the constraints into account in a similar way:

$$pu_{\boldsymbol{g}_j}(\boldsymbol{X}) := \min\left\{\frac{-\underline{\boldsymbol{g}_j}(\boldsymbol{X})}{w(\boldsymbol{g}_j(\boldsymbol{X}))}, 1\right\}, \quad pu(\boldsymbol{X}) := \prod_{j=1}^{r} pu_{\boldsymbol{g}_j}(\boldsymbol{X}).$$

(where $\boldsymbol{g}_j$ is the interval inclusion function of the jth constraint). The pu quantity measures the relative position of 0 within the inclusions of the constraint functions, i.e. the feasibility of the box $\boldsymbol{X}$. Finally, the heuristical decision index for constrained problems is formalized by $pup(\hat{f}, \boldsymbol{X}) := pu(\boldsymbol{X}) \cdot p(\hat{f}, \boldsymbol{X})$. We can conclude that if the pup value for a given box is high, then the box should be preferred for an early selection (interval selection step), or it is advisable to split it into a higher number of subboxes (subdivision step).

In the theoretical part of the talk we present convergence properties of the B&B algorithms using the new interval selection criteria. As a conclusion, we can state that a suitable choice of the $\hat{f}$ value is essential to reach the global convergence.

In the numerical tests we introduce the efficiency of the new heuristic rules on three different types of problems: the first is the problem class of the obnoxious facility location model [4]. For such a problem our goal is to place an unpleasing object into a region by considering the disappointment of the inhabitants and the geographical possibilities. The second part of the test problems came basically from unconstrained global optimization; we have selected some harder problems, e.g. Hartman-6, Goldstein-Price, Levy-3, Ratz-4 and EX2 (for the definitions see [2, 9, 10]) and completed them with sets of randomly generated linear and quadratic constraints.

In the third part of the numerical investigation we discuss the 'packing circles into a unit square' problems [5, 6, 8]. For such a problem instance (specified by the number of the circles, n) we want to determine the largest value of r for which the given number of circles having the uniform radius r can be placed into the square without overlapping. We present an improved algorithm giving verified solutions, where the verification can be made in both the local and the global sense.

The main consequence of our investigations is that the new type of interval selection criterion significantly improves the efficiency in terms of both the running time and the memory complexity. In addition, the largest improvements were achieved on the hardest problem instances.

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