Optimal substructures in optimal and candidate circle packings

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The densest packing of equal circles in a square problem is a well-known challenge of the discrete and computational geometry. Using a computer-aided method, up to 27 circles could be found the optimal solutions. Based on only mathematical tools, without computer, the best arrangement of 36 circles is also known. For higher number of circles we have only candidate packings up to 200 and for some sporadic values [3]. The candidate packings are the best known arrangements without the proof of optimality.

The greatest difficulty to solve this problem is to find the structures of optimal packings. Some previous articles have already investigated repeated patterns of packings [1, 2] and at the previous $(CS)^2$ conference we have given a classification on structures based on minimal polynomials.

In this talk I would like to show some interesting situations when the optimal and candidate packings contains optimal substructures. These substructures are the locally densest packings inside a packing. The shapes where their density are maximal can be different (e.g. square, triangle or circle).

Sometimes the exact value of the radius of circles in the optimal packing is unknown. In this case, it can be give a minimal polynomial with the first positive root of the correct radius (or the minimal distance between the points). To determine these polynomials in some cases are trivial but sometimes very hard. The calculation usually based on the theory of Groebner bases. In my approach I used generalized minimal polynomials of the substructures to calculation the minimal polynomial for the total structure.

References

[1] P. G. Szabó, Some New Structures for the "Equal Circles Packing in a Square" Problem, Central European Journal of Operations Research 8:79-91, 2000.

[2] P. G. Szabó, T. Csendes, L. G. Casado and I. García, Packing Equal Circles in a Square I., Optimization Theory: Developments from Mátraháza, Kluwer Academic Publishers, pp. 207-224, 2001.

[3] P. G. Szabó and E. Specht, Packing up to 200 Equal Circles in a Square (in preparation).