A verified computational technique to locate chaotic regions of a Hénon system

Balázs Bánhelyi

We present a computer assisted proof of the existence of a horseshoe of the 5th iterate classical Hénon map $(H(x,y) = (1 + y - \alpha x^2, \beta x))$. An earlier, published theorem [3, 4] gives three geometrical conditions to be fulfilled by all points of the solution region, given by 2 parallelograms. We analyze these conditions separately and in case when all of them hold true, the proof is ready. The method applies interval arithmetic and recursive subdivision. This verified technique proved to be fast, and we can use it in a framework program. To find a region that fulfills the respective conditions, the program combines a global optimization procedure and our interval arithmetic [2] based checking technique described earlier. The algorithm obtains the Hénon map and the parallelogram parameters and checks whether these fulfill the c onditions. If not then it provides a penalty for this structure for the optimization procedure. The penalty is zero if the structure doesn't brake any conditions. In this way we have obtained an optimization problem with six parameters. If the program finds the optimum and it is zero, then the search is successful, and we were able to locate a region where the investigated Hénon map instance has a chaotic behaviour. The obtained coordinates of the lower parallelogram vertices for the Hénon transformation parameters $\alpha = 1.939838, \beta = 0.39146881$ are

$$x_a = 0.33298647, x_b = 0.49115518, x_c = 0.50960044, \text{ and } x_d = 0.59020179$$

with $y_0 = 0.01, y_1 = 0.28,$ and $\tan\phi = 2.0.$

In addition to the above result, we have extended the result of [4] in the sense that in stead of their Hénon parameter values of $\alpha = 1.4$ and $\beta = 0.3$, we have determined a set of those parameter values that cause the chaotic behavior for the H^7 transformation with the same parallelograms. The obtained intervals were $\alpha \in [1.377599, 1.401300]$ and $\beta \in [0.277700, 0.310301]$. The technique with which this result was obtained is an earlier interval optimization procedure able to solve tolerance optimization problems [1]. The author is grateful to Barna Garay (BME, Budapest) and Mihály Görbe (GAMF, Kecskemét, Hungary) for their contribution and support.

References

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