Holonomy Decomposition of Finite State Automata

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The hierarchical algebraic decomposition of finite state automata is not just an important part of theoretical computer science but also has many possible applications in all different fields where we deal with hierarchical models of systems: physics [13], software-development [9], artificial intelligence [10], evolutionary biology [12], etc. We are concerned mainly with the formal models of understanding from the artificial intelligence intelligence point of view [10] where the decomposition is regarded as an easy to manipulate (via the coordinates) model of the original automaton.

The original Krohn-Rhodes Theorem has many different formulations (e.g. [7, 8, 6, 4, 5, 11, 1]). The results of the first computational implementation [2] of the theorem showed that in the terms of the length of the decomposition the holonomy method [6, 4, 1] performs better than other algorithms: the number of hierarchical levels is less than the number of states in the original automaton. The improved algorithm [3] for the holonomy decomposition renders the computational investigation of the hierarchical structure of finite state automata feasible.

Here we examine the hierarchical decomposition of different types of finite state automata in terms of their holonomy structures: the pre-order subduction relation on the state set, the size and the number of equivalence classes of the mutual subduction relation, the tiling structure and the holonomy groups. The types of automata are those which have a corresponding characteristic semigroup that is cyclic, full, left or right simple, aperiodic, group, permutation-reset, definite, or reverse-definite. The clarification of the relationship between different automata and their hierarchical decomposition given by the holonomy algorithm helps the elaboration the concept of a formal model of understanding. Beyond the theoretical insights provided by this investigation the results also help in improving the decomposition algorithm itself as special properties can be used when restricted to one particular class of automata.

Further details and the software can be find at `http://graspermachine.sf.net`.

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