Integer Merge Model Representation of the Graph Colouring Problem

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The vertex graph colouring problem or put shortly the Graph Colouring Problem (GCP) plays important role in graph theory. The GCP arises in a number of applications for example timetabling, scheduling or register allocation. It deals with assigning colours to the vertices of an undirected graph such that adjacent vertices do not get the same colour. The primary objective is to minimize the number of colours used. The fewest number of colours necessary to colour the vertices of a graph is the chromatic number. Finding the chromatic number of a graph is an NP-complete problem [3]. Thus, one often relies on heuristics to compute solution or its approximation.

Graph colouring algorithms make use of adjacency checking during colouring, which plays a significant role in performance. The number of checks depends on the representation of the problem and the algorithm based on it. The Integer Merge Model (IMM) introduced here directly addresses these issues. This is a generalization of the efficient Binary Merge Model [2], therefore it gives a good representation of the GCP. Embedding it into a graph colouring algorithm we are able to reduce the number of adjacency checks and define useful heuristics using the model provided information.

Generally, when assigning a colour to a vertex all adjacent vertices must be scanned to check for an equal colouring, i.e., adjacency checks need to be performed. Thus, we have to perform at least as many checks as the number of coloured neighbours and at most as the number of vertices in the graph. In the new model, this number of checks is between one and the number of colours used up to this point. It thanks to the model induced hyper-graphs.

To show performance issues and the ability of defining useful heuristics on the model, IMM was embedded into the DSatur of Brélaz [1], which is a standard among the graph colouring algorithms. An empirical comparison is made between DSatur with and without IMM on a standard suite of problem instances. The results show that IMM gives a compact, general and powerful representation of the problem.

References

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