M/G/1 Queuing System with Two Types of Vacation

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To determine the equilibrium distribution of different queuing systems, the embedded Markovchain technique is often used, which leads to the PollaczekKhinchin formula, the generating function of ergodic distribution. From mathematical point of view this gives the exact solution of the problem, and probabilities can be derived from it by means of differentiation. This seems to be the most natural way, but it gives very complicated results. These difficulties induced the search for other methods.

In [2] Briére and Chaudhry considered bulkarrival systems, and found a recursive algorithm for different service time distributions. The inversion of generating functions are realized by comparing the coefficients of the corresponding powers of z. Their work also includes both sample numerical results and easily implementable algorithms. Lakatos used another approach, he described queuing systems with the help of Kovalenko's piecewise linear processes [1]. This makes it possible to calculate the desired probabilities on the basis of the mean length of a busy period and the mean value of time spent in different states. In [4] he gave recursive formulae for the equilibrium distribution of the ordinary M/G/1 system. He also found such formulae for the system where after each busy period there is a vacation, and a cycle ends when there are no entering customers during a vacation. In [5] he generalized formulae for bulkarrival systems. In [6] Lakatos investigated such a queuing system where the arrival of the first customer initiates a vacation, and service can only start when the system is prepared for it during the vacation. The abovementioned method made it possible to give recursive formulae for this type of system. In this paper we are going to generalize results for an M/G/1 system, where there are vacations both at the beginning of the service and after each busy period. We give ergodic distribution by calculating the mean value of length of a cycle, and finding recursive formulae for the mean time spent in different states.

References

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