Unification for Effective and Finite Semantic Tableaux in First-order Logic: the SOFIA Prover

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The problem of automated theorem proving in first-order logic has been tried to be handled by the use of semantic tableaux [1], among other tools. Since it is an algorithmically insolvable problem by nature, tableaux meet the problem of the γ-formulae, which is algorithmically insolvable, too. By the use of free-variable semantic tableaux, this insolvability can be overridden. As detailed in [2], even the application of the Most General Unifier (MGU) Atomic Closure Rule for free-variable tableaux effectuates a non-deterministic and ineffective tool for theorem proving, furthermore its execution may stuck in an infinite loop. In our paper, a straightforward and very effective technique is proposed for free-variable tableaux. The main question, namely which MGU should be applied, is replied by the construction of a heuristic measure, which is used for ranking MGUs generated for a branch of a given tableau. The heuristics is a kind of a hierarchical one based on the number of branches and the number of bound variables. Whilst a hardly usable check for depth-limit was performed in [2], our technique proposes a very simple trick in order to avoid infinite loops. In the paper, we propose the Extended Unification Algorithm, which can produce an MGU for formulae (not only for terms), i.e., it reveals if each of them can be transformed to a same formula by the application of a substitution on their parameters. The Extended Unification Algorithm is a generally used tool for all the aforementioned techniques, i.e., not only for the generation of MGUs but also for avoiding infinite loops.

Besides proving a theorem, the proposed prover is especially useful in answering the following question: for a formula A, if a substitution θ exists where $A\theta$ is a theorem. It can be seen, theorem proving is only a special case of this issue, namely the case when $\theta = \emptyset$. Accordingly, the proposed prover could answer Prolog-like questions like `?- A(X1, ... ,Xk)`, where A is an arbitrary first-order formula (even a compound one) with the parameters $X_1, \ldots, X_k$ $(k \geq 0)$. This kind of functionality can be easily achieved by the extension of the heuristics on MGUs, namely by making the heuristic measure reactive to the number of free variables.

The proposed prover has been implemented and named SOFIA.

References

[1] Raymond M. Smullyan, “First-Order Logic”. Springer-Verlag, 1968.

[2] Melvin Fitting, “First-Order Logic and Automated Theorem Proving”. Springer-Verlag, 1996.

[3] Francis J. Pelletier, “Seventy-Five Problems for Testing Automatic Theorem Provers”, *Journal of Automated Reasoning*. 1986, Vol. 2, p. 191-216.