

Primitive Words and Permutations

Benedek Nagy and Szilárd Fazekas

In this paper we will analyse some interesting languages such as the languages of primitive words, Lyndon-words and their commutative closure. The language of primitive words has been the subject of numerous studies. It is the language of the words that are not a proper power of another word. Basically the most important question about it is whether it is or is not a context-free language. The conjecture is that it is not context-free, but in this paper our analysis is going to another direction. We will analyse some permutability properties of this language. The languages of Lyndon words contains all primitive words which are minimal under cyclic permutation. It is known that this language is a non-context-free, context-sensitive language. In this paper we will consider the commutative closure of these languages, containing all permutations of the letters of their each word. We can consider this language as a multiset language containing Parikh-vectors of the original languages. In this paper we investigate another languages based on primitive words and permutations. Namely, we analyze the language of the words which are primitive and every permutation of them is primitive, in other words the multiset languages which correspond to the maximal commutative language containing only primitive words. A multiset language that contains only primitive words consists of Parikh-vectors made out of relative primes. Given such a language it is interesting to check out how many of the words among those that have the same Parikh-vector are Lyndon words i.e. minimal under cyclic permutation. Another question being analyzed in the talk is whether this multiset language is linear or semi-linear in Parikh-sence.

Note, that the multiset languages play important role in membrane computing and in some other non-classical processing systems.

Some results concerning permutation loops applied to Lyndon words of the forementioned language are also given.