On the Ambiguity of Reconstructing Decomposable hv-Convex Binary Matrices

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The reconstruction of binary matrices from their projections is a basic problem in discrete tomography. Reconstruction algorithms have a wide area of applications in graph theory, non-destructive testing, statistical data security, biplane angiography, crystallography, radiology, image processing, and so on. For practical reasons the projections can be taken only from few (usually at most four) directions. This often leads to ambiguous or/and NP-hard reconstruction which is inappropriate for applications. One commonly used technique to reduce ambiguity and to avoid intractability is to suppose having some a priori information of the matrix to be reconstructed. In this paper we assume that the matrix to be reconstructed is hv-convex and decomposable. First, we give a construction to prove that the use of only two projections is not sufficient to eliminate ambiguity, that is, for some inputs there can be exponentially large number of hv-convex decomposable binary matrices having the same horizontal and vertical projections. In the case of four projections we are faced the following problem. Although using four projections all the binary matrices of this class with the given projections can be reconstructed in polynomial time [1] the class of decomposable hv-convex matrices is not explicitly defined. In more detail, one criteria of decomposability is that the components of the binary matrix are uniquely reconstructible from their horizontal and vertical projections. However, when reconstructing hv-convex matrices it cannot be decided in advance whether this criteria is satisfied. This sometimes does not lead the reconstruction algorithm to fail, i.e., the reconstruction algorithm possibly gives a solution even if one of the components is not uniquely determined. We do experiments to investigate the possibility that a component (which is an hv-convex polyomino) is uniquely determined by its horizontal and vertical projections. We also study how the knowledge of a component's third projection effects on the ambiguity.

References

[1] P. Balázs. A decomposition technique for reconstructing discrete sets from four projections, *Image and Vision Computing*, submitted.