

A linear programming background for the R_{HFF}^∞ upper bound proof

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In the last 30 years intensive research was carried on the behavior of different kinds of bin-packing (BP) algorithms, where the goal is to put different tall items into the minimum number of unit height bins. There are several versions of BP algorithms, like on-line, off-line, d-dimensional and k-repacking problem.

In the on-line BP problem you have to put the items one by one into their final position without knowing the size of the remaining items. In the d-dimensional BP problem the items have d dimensions, and you have to pack them into d-dimensional one unit „width” bins.

Our research focused on the worst case scenario of the BP algorithms. The asymptotic worst case ratio of algorithm A (R_A^∞) shows us, for any input series of items, how many times more bins are needed in the worst case to pack the items with algorithm A , comparing to the optimum packing requires.

In 1982 Chung, Garey and Johnson introduced their off-line two-dimensional bin-packing algorithm, called the Hybrid First Fit (HFF)[1] algorithm. In their work the authors proved the lower bound of R_{HFF}^∞ to be at least $\frac{182}{90}$ by giving a special input series of items. They also proved the upper bound to be at most $(\frac{191.25}{90})$. We expect the exact value of R_{HFF}^∞ to be somewhat less than $\frac{187}{90}$.

In the upper bound proof the authors used a „horizontal lines through the bins” method, where they had 5 item classes and they crossed the bins with lines at different positions and counted up all the items more times. Unfortunately in [1] there is no word about how the authors found the proper number and position of the crossing lines.

We have developed two Linear Programming (LP) models, where solving these LPs give the number and position of the crossig lines and we get exact the same numbers that Chung, Garey and Johnson found in their work. The question is if we can use this LP model to generate the number and position of the lines, than are we able to improve the R_{HFF}^∞ by using more and more item classes?

References

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