The Chooser-Picker 7-in-a-row-game

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Our main objective is to show how the Beck's conjecture intertwines the Chooser-Picker games with the k-in-a-row games. The k-in-a-row game deserves consideration by itself, because the last result of this topic was the famous theorem about the 8-in-a-row game (it is a blocking draw on the infinite chess-board) is more than 27 years old, see [3, 4]. Since then a lot people tried to prove similar theorem for the 7-, or 6-in-a-row-game, but up to now without success. We sketch a possible way to show that (assuming perfect play) the 7-in-a-row is also a blocking draw.

Given a hypergraph $(V, \mathcal{F})$ there are a number of games that can be played on it. In the *Maker-Maker* version, the first player and the second player take the elements of V, and the winner is who gets all elements of an $A \in \mathcal{F}$ first. In the *Maker-Breaker* version Maker wins by occupying all elements of an $A \in \mathcal{F}$, while Breaker wins by preventing Maker in doing so. Finally in the *Picker-Chooser* version Picker selects two vertices of V, Chooser takes one of those, and then the other of course goes back to Picker. Chooser wins if he occupies a whole winning set, while Picker wins if he can prevent this. When V is odd, the last element goes to Chooser.

While the Chooser-Picker games are interesting on their own, these are also useful tools to understand positional games better, see details in [2, 1]. Namely, from the result of a Chooser-Picker game we have an insight to the result of the Maker-Breaker version: these coincide several times, and it seems that Picker is *always* better off than Breaker.

In this work we prove that Picker wins the Chooser-Picker version of the 7-in-a-row game. To prove the theorem, we exhibit an appropriate tiling of the board. The tiling is constructed in such a way that if Picker wins an auxiliary game on each tiles then Picker wins the original 7-in-a-row game. The proof of this case is a medium size case study.

References

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