Employing Pythagorean Hodograph Curves for Artistic Patterns

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In the paper we will present our design element creator tool for the digital artist. The purpose of our tool is to support the creation of vines, swirls, swooshes and floral components. To create gentle and pleasing curves we employ Pythagorean hodograph (PH) quintic curves to join a hierarchy of *control circles* defined by the user. The control circles are joined by spiral segments with at least G^2 continuity, ensuring smooth and seamless transitions. The control circles give the user a fast and intuitive way to define the desired curve. The resulting curves can be exported as cubic Bézier curves for further use in vector graphics applications.

Introduction and Background

Floral elements, vines, tangled spirals and similar features are among the most popular design components. These components could be found in traditional ornamental and contemporary abstract designs as well. There have been several efforts to automate aspects of the artistic process of creating such designs and ornaments. Since the fundamental work of Prusinkiewicz and Lindenmayer [1] L-systems has been used widely to generate flowers and flower like patterns. Wong et al. [2] presented a system to automatically generate space-filling floral ornaments. Their system uses proxy objects during generation what could be replaced by arbitrary elements created by any means. Xu and Mould's *Magnetic Curves* [3] are more closely related to our work. They focus on the creations of the curves themselves, but their method uses a discrete time-step approach, and they commit the problem of creating smooth curves to approximation routines.

We based our work on the same assumption as Xu and Mould, that is, a pleasing curve has a smooth curvature. Our goal is to make a tool for the digital artists to create such curve or sequence of curves with ease. Out method uses cubic and quintic splines to generate resolution independent curves, which can be used themselves, serve as a skeleton of a design, or act as a path for strokes or objects. To support the widest range of third party tools possible, the generated curves can be exported as cubic Bézier splines.

PH curves introduced by Farouki and Sakkalis [4] have very favourable properties, most importantly it is possible to define spiral segments using PH quintic curves whose curvature changes monotonically with arc-length. These curves have been used in highway, railway and robot trajectory design. Now we would like to show the efficiency of these curves as design elements as well.

Our tool In our tool the user defines a hierarchy of control circles to create her design. If it is possible the system automatically connects a circle to its ancestor with an appropriate curve.

Our system supports circle-to-circle S-shaped curves, circle-inside-circle transitional spiral segments, and circular arcs approximated by Bézier curves joining the two preceding. During design additional properties are specified for each control circle.

Both for the circle-to-circle S-curves and circle-inside-circle spiral segments Pythagorean hodograph quintic curves are used. These curves have been defined to have G^2 contact to their control circle. Therefore, each incoming curve can have G^2 contact with any outgoing curve of the same circle. Two non-touching, non-overlapping control circles are connected by an S-shape, if within a certain threshold. The derivation for the control points of the S-curves is following the work of Walton and Meek. The positions and radii of the circles define the shape of the S, the only additional user input required is whether the shape should be mirrored or not.

A fully contained circle is joined to its ancestor with a spiral segment, if such transition is possible. The conditions and the derivation of the control points are given in Habib and Sakai's [5] work. The radii of the circles define the range of allowed distances between the centres. For now, our software chooses the smallest possible distance. For a given pair of radii and distance, the spiral segment is uniquely defined, thus the radii and centre-to-centre distance dictates the positioning of the smaller circle. If the creation of the transition curve is possible, only the radius of the smaller circle is used, and its centre is repositioned as defined by the algorithm. Similarly to the S-curves, a mirroring property can be defined. An additional parameter of a spiral segment is its starting point, defined in degrees on the arc. For new segments, this is automatically calculated, so the new curve continues the ancestor's incoming curve, if it exists.

Because the computations of the control points in both cases are quite involved, especially for the second case, several numerical algorithms were required in the implementation. In the derivation of the curve parameter theta for the first case, Halley's method is used, since the simpler Newton's method proved to fail on several occasions. However, Newton's method is sufficient for the second case, during the calculation of the valid range of centre distances. Unfortunately, none of these methods were useable for determining the theta value in this case. The derivative of the concerned function is too complex to be useable. Nevertheless, because the domain of the possible solutions is known, a simple bisection method has proven adequate.

The conversion from Pythagorean hodograph quintic to cubic Bézier are done by elementary degree reduction, with endpoint correction. This is sufficient, because PH quintic curves have very similar geometric properties to cubic Béziers.

References

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