Circle Covering and its Application for Telecommunication Networks

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The circle packing problem has attracted much attention in the last century, and a variant called packings of equal circles in a square receives attention even nowadays. The objective of it is to give the densest packing of a given number of congruent circles with disjoint interiors in a unit square.

However, its dual problem, the circle covering has not been exhaustively studied so far. We aim to find the "sparsest" covering of the unit square with a given number of congruent circles with overlapping interiors allowed. By sparsest we mean the total covering of the square with congruent circles of minimal radii.

The main difficulty of the problem is the uncertainty of our computations caused by the finite precision of computers. To overcome this we have used interval arithmetic to test if a given setting of circles covers the unit square completely. We developed a branch-and-bound–based method for the previously mentioned testing function. We examined our method in a parallel environment, when the phases of our method can be executed concurrently on several CPU cores.

To test the efficiency of our method we also applied it to a telecommunication-related problem: We would like to find an optimal covering of Hungary with TV-stations for terrestrial signal given the positions of the broadcasting antennas. The signals cover a circular area around the broadcasting stations, and our aim is to minimise the sum of these areas, which is proportional to the power fed into the stations. In this case the radii are allowed to be of different size, and to determine the optimal solution we applied a B&B–based reliable optimisation method.

Keywords: Circle covering, interval arithmetics, reliable computing, parallel computing.