Characterization of Semi-CNS Polynomials

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Let $P(x) = c_0 + c_1 x + ... + c_{n-1} x^{n-1} \in \mathbb{Z}$ and $D = \{0, 1, ..., c\_0 - 1\}$. The polynomial $P(x)$ is called CNS-polynomial if every coset of the factor ring $\mathbb{Z}[x]/P(x)\mathbb{Z}[x]$ has a member of form

$$\sum_{h=0}^{\infty} d_h x^h, \tag{1}$$

with $d_h \in D, h = 0, 1, \ldots$ and such that only finitely many d_h are non-zero. This concept generalizes the negative-base radix representation of integers. It was introduced and studied in [2]. The characterization of CNS polynomials already for degree three is complicated, as indicated in [3]. It is still unsolved.

Burcsi and Kovács [1] called $P(x)$ a *semi-CNS polynomial* if the finite expansions (1) form an additive semigroup. This is a generalization of the usual radix representations of natural numbers. They were able to prove some sufficient properties for $P(x)$ being a semi-CNS polynomial. Moreover they generalized Brunotte's algorithm for semi-CNS polynomials.

In this talk, which is based on a joint work with A. Pethő we give a complete characterization of cubic semi-CNS polynomials. More precisely, in all those polynomials, which do not satisfy the condition given by Burcsi and Kovács, are not semi-CNS, so all those cubic monic polynomials which has a negative coefficient in addition to the constant one are not semi-CNS. To prove this we present to each polynomial a cycle.

References

[1] Péter Burcsi and Attila Kovács: *Exhaustive search methods for CNS polynomials*, Monatsh. Math. 155, 421-430 (2008).

[2] S. Akiyama, T. Borbély, H. Brunotte, A. Pethő, and J. Thuswaldner: *Generalized radix representations and dynamical systems I*, Acta Math. Hungar., **108 (3)** (2005), 207-238.

[3] S. Akiyama, H. Brunotte, A. Pethő, and J. M. Thuswaldner: *Generalized radix representations and dynamical systems II,* Acta Arith., **121** (2006), 21-61.