A DEVELOPMENT MODEL OF PLACE-VALUE AND BASE-10 SYSTEM

Moritz Herzog
University Duisburg-Essen

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The current situation: The base-10 system is by far the most used and important number system. At the same time the place-value system is a main obstacle for many pupils. Some time ago several development models were published (*Resnick* 1983, *Cobb* & *Wheatley* 1988, *Ross* 1989). These models have not been validated by broad empiric research. In addition, they are little mentioned in scientific publications. As a consequence, there is no validated and commonly accepted development model for the base-10 system at this time.

Method: In our conceptual model for the development of the base-10 system, we postulate some hypothetical levels, building upon each other hierarchically. Our model is based on the ideas of the existing models and on our data, captured in some pilot studies including base-10 tasks with different difficulties with at least 9,500 students in grades 3 to 7 (2010–2013). In this poster we present the data of our last pilot study with 1.267 children from grades 4 to 5 in different parts of Germany.

The model: The competence-model of the place-value system was tested in a Rasch-analysis, which confirmed the hierarchical structure of the test with five levels of ability, of which two are divided in two sublevels each. The model holds all criteria of the Rasch-scaling. Only one item has infit-MNSQ-value between 0.7 and 1.3 (the others between 0.8 and 1.2).

Level I: Multi-digit numbers are seen as entities not decomposable into smaller units.
Level II: The names of the place-values are known and can be named, but they are not connected by any concept of ten (*Cobb* & *Wheatley* 1988).
Level IIIa: The possibility to bundle ten ones to one ten is known; bigger units cannot be processed.
Level IIIb: Unbundling of one ten into ten ones is understood.
Level IVa: The knowledge of bundling is enhanced to units bigger than ten: Ten hundreds make one thousand and so on.
Level IVb: Units bigger than ten can be unbundled.
Level V: Tens, hundreds and so on serve as "milestones" on the mental number line helping to orientate on it (cf. *Resnick* 1983).

Importance and further research: This model has to be validated by further tests. Also, there is still some uncertainty regarding the order of Levels IIIb and IVa as well as regarding when the use of material and its imagination is no longer required (cf. *Cobb* & *Wheatley* 1988). A validated development model of the base-10 system can be used in diagnostic tests as well as in training programs in order to improve the quality of mathematical teaching.

References

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University Duisburg-Essen.