On Some Subsystems of Interval-Valued Logic

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The non-classical logic we consider here differs from classical propositional logic mainly in the interpretation function. A computing device, based on this idea can be quite powerful as shown in [3, 4, 5], also an application of this paradigm can be found in [6]. In classical propositional logic, an interpretation maps each propositional variable an element of the set $\{0,1\}$. In the case of many-valued logics, variables can be assigned values from different sets (e.g. $\{0,\ 1/2,\ 1\}$, $\{1,\dots,n\}$, or even $[0,1]$). In the case of interval-valued logic (see for example: [1, 2]), a union of subintervals of $[0,1]$ is assigned to each variable. We also require all of these subintervals to be in the form $[a,b)$. In other words an interval-value is a set of ordered pairs $(a_{i,1},a_{i,2})$ $(i=1,\dots n)$ such that $0 \leq a_{i,1} < a_{i,2} < 0$, and $a_{i,1} \leq a_{j,1} < a_{i,2}$ is not possible for any distinct $i,j \in \{1,\dots,n\}$. Logical operators negation, conjunction, disjunction and implication are defined as $\overline{A} = [0,1) \setminus A$, $A \cap B$, $A \cup B$, $\overline{A} \cup B$ respectively, where A,B are interval-values. Some non-logical operators are also defined in [2], here we only mention $*$, the product operator that is

$$A * B = \bigcup \{[a_{i,1} + b_{j,1}(a_{i,2} - a_{i,1}), a_{i,1} + b_{j,2}(a_{i,2} - a_{i,1})) \mid 1 \leq i \leq n, 1 \leq j \leq m\},$$

where A and B consists of intervals $[a_{n,1},a_{n,2})$ $(1 \leq n \leq k)$ and $[b_{m,1},b_{m,2})$ $(1 \leq m \leq l)$ respectively. We note that this product is not commutative, but associative. An additional operator, based on Kleene's strict implication will be defined. One possible usage of this operator is to represent the semantic consequence relation.

We will investigate subsystems of interval-valued logics, where each value consists of only one interval. Thus the following two main cases are considered:

1. Each value starts from a designated end of $[0,1)$. We have three sub-cases:

   All interval-values are in the form

   (a) $[0,a)$ where $0 < a \leq 1$,

   (b) $[a,1)$ where $0 \leq a < 1$,

   (c) $[0,a)$ or $[b,1)$ where $0 < a \leq 1$ and $0 \leq b < 1$.

2. Each value is in the form $[a,b)$, where $0 \leq a < b \leq 1$.

In each of these cases, some logical operators must be redefined in order to have a closed system. We will prove logical equivalences and show, that in all of these cases $\neg X \vee X$ is not necessarily a tautology. Thus some of them can be considered as intuitionistic logics.

References

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