Wavelets and rational function methods in ECG signal processing

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The electrocardiograms (ECG) represent the electrical activity of the heart related to different phases of the muscular contraction. These electrical changes can be measured on the surface of the skin by electrodes. An ECG signal records the potential difference between these electrodes. The electrocardiogram consists of five standard waves labelled with the letters P, Q, R, S, T. The amplitudes, durations and shapes of these waves carry important diagnostic information about the patients. There is a wide range of recent applications of digital signal processing including detection, analyzation and compression.

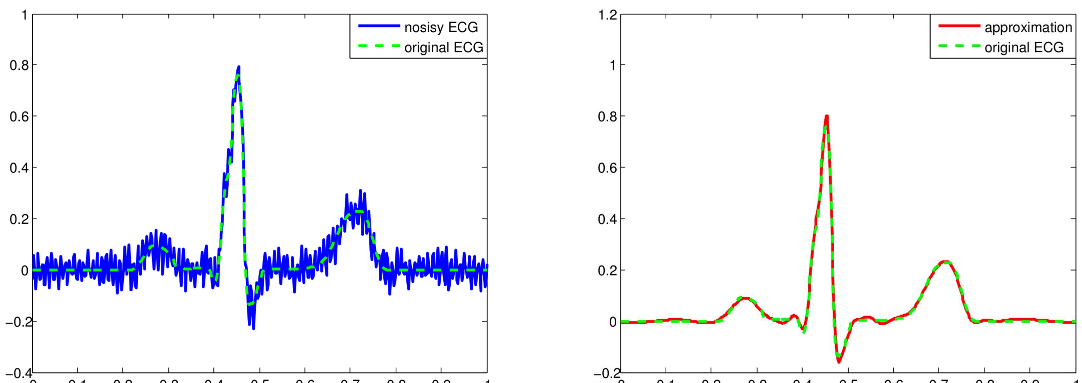
Lately, we used rational functions to model the electrocardiogram [3]. In order to outline this method, let us take the elementary rational functions on the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$

$$r_a(z) := \frac{1}{1 - \overline{a}z} \qquad (|z| < 1) ,$$

where the pole is $a/|a|^2$. a itself is called the inverse pole of r_a . Then we take linear combinations of powers of elementary functions and consider them on the unit circle. We use these functions to model the ECG signal f as follows

$$f \approx \sum_{n=1}^{N} \sum_{j=1}^{n_k} c_{n,j} \cdot r_{a_n}^{j} \qquad (N, n_k \in \mathbb{N}),$$

where the $c_{n,j}$'s are complex coefficients. Three different poles turned to be appropriate in most cases [2]. In the present work we compare the widely used wavelet-based techniques [1] with those of rational functions [3]. We compared these methods in various aspects.



(e) Synthetic ECG signal with an 50Hz power line interference.

(f) Denoised ECG curve smoothed by basic rational functions.

Figure 2: Rational function approximation of a synthesized electrocardiogram.

Firstly, we compare the rate of compression by rational functions in contrast with the Daubechies and Morlet wavelets. Furthermore, we want to use an objective measure of level of compaction and distortion rate. For this reason, synthesized ECG signals [5] were used to test these methods. In this case, we can generate electrocardiograms with different diagnostical and geometrical features, but we can get the original signals in analytic forms as well. So, both compression and distortion rate can be measured by using different types of error measures, such as percentage root mean square difference (PRD) and weighted diagnostic distortion (WDD) [6]. By taking advantage of the analytic form of the original signal, one can test the sensitivity of these methods by changing the signal-to-noise ratio (SNR). On Figure 2(f) one can see the reconstructed signal marked by red line. The original ECG curve has 512 sample, but we used only 16 complex coefficients and 3 complex poles for the representation.

Secondly, a related problem is the filtering property. Namely, these methods can be used for denoising ECG signals. Different types of noise such as muscle noise, baseline drift, etc. were used to test the smoothing ability of rational functions in contrast with Daubechies and Morlet wavelets. During filtering it is important to keep the most significant ECG features by tempering the distortion rate of the process. An example for rational filtering is given in Figure 2.

Finally, we propose a wave detection algorithm based on the rational function approximations. This method separates one period of an electrocardiogram into three main parts that are designated alphabetically as P, QRS and T waves. On one hand, it is a rough approximation of the original signal, because we use only the most significant terms from the projection (2). On the other hand, the onsets and the offsets of the main lobes of the ECG curve can be predicted well. Furthermore, both synthesized electrocardiograms and real ECG databases such as physionet [4] were used to test and compare the accuracy of this wave detection algorithm with wavelet-based methods. These databases are also annotated, so the test results can be easily achieved.

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