Geometric Newton-Raphson Methods for Plane Curves

Gábor Valasek, Levente Sallai, András Jámbori, Júlia Horváth

Plane curves play an important role in computer graphics, geometric modeling, GIS systems, and other applications. Queries on these curves, such as findig the closest point of a curve to a given point of the plane, or finding the intersection of two plane curves, often require the solution of a system of non-linear equations or finding the roots of a non-linear function. The Newton-Raphson method is a popular choice for solving these problems iteratively.

Kallay presented a geometric Newton-Raphson strategy in [1]. Each new guess was computed by solving a geometric approximation of the problem at the current guess. The geometric approximation consisted of substituting the curve with its osculating circle, and using the solution of the query on the osculating circle to choose the new guess.

Our paper presents a secant method-like modification of Kallay's algorithm and compare it with the classic and Kallay's geometric Newton-Raphson methods in point-curve distance and curve-curve intersection queries, with emphasis on computational cost and robustness issues. The osculating circle is approximated by fitting a circle to points on the curve, avoiding the evaluation of the curve's derivatives. Another modification of Kallay's method, that uses osculating parabolas and their approximations instead of osculating circles, is also presented and evaluated.

Acknowledgements

This research was supported by the project TÁMOP-4.2.1/B-09/1/KMR-2010-003 of Eötvös Loránd University.

References

[1] Kallay M. A geometric Newton-Raphson strategy. Computer Aided Geometric Design, Volume 18, Number 8, October 2001 , pp. 797-803(7)