

Generalized Haar systems toolbox for MATLAB

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The Haar system or Haar wavelet is a well-known mathematical tool proposed by the Hungarian mathematician Alfréd Haar with an abundance of applications in the field of signal and image processing. The system forms an orthonormal function basis in $L^p[0, 1)$ ($1 \leq p < \infty$), and the Fourier series of any continuous function with respect to this system converges uniformly to it. Furthermore the structural simplicity of the Haar wavelet makes it ideal for application in computer software.

Considering these notable properties it is desirable to generalize the Haar wavelet and construct different systems with similar properties [1]. A key concept in this regard is the notion of product systems. A well-known example is the relationship of the Haar, Rademacher and Walsh functions. Using this concept, it is possible to construct orthogonal systems of rational functions analogous to the Haar wavelet using the Malmquist-Takenaka system as a starting point [2] (see Figure 2). An advantage of rational functions over the Haar wavelet is that the former are analytic, while the latter is not even continuous.

In this paper we present a MATLAB toolbox for working with generalized Haar systems. The toolbox contains efficient implementations of signal transformation and reconstruction algorithms using various function systems. Two dimensional variants for image processing are also presented. The library is based on the RAIT rational approximation and interpolation toolbox [3].

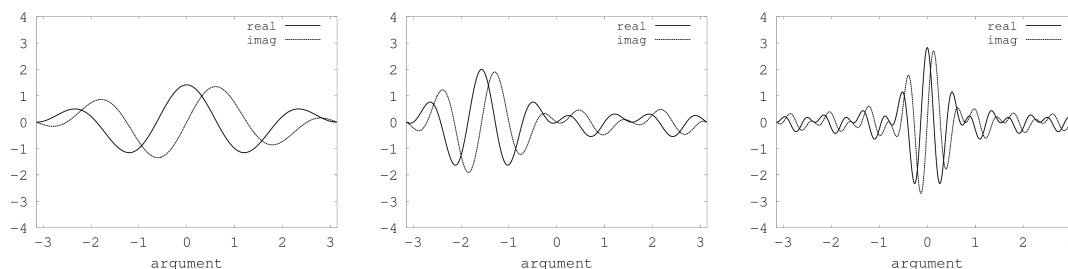


Figure 2: Rational Haar-like functions on the complex unit circle.

References

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- [2] F. Schipp. Rational Haar systems and fractals on the hyperbolic plan, *Sacks Memorial Conference*, Szentgotthárd, 2003, Oskar Kiadó.
- [3] P. Kovacs and L. Locsi. RAIT: The rational approximation and interpolation toolbox for Matlab, *Telecommunications and Signal Processing (TSP) 2012 35th International Conference*