A new Approach to Fuzzy Control using Distending Function

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Abstract: The paper presents a novel design for fuzzy logic control using the Distending Function (DF) as a membership function. The proposed design is close to human input and can be transformed into fuzzy model for stability analysis using conventional control theory. The design procedure is simplified by employing fuzzy arithmetic and properties of this new type of membership function. Its compatible with the existing operators system and can be used with already developed fuzzy models to achieve greater flexibility in design process.The effectiveness of the proposed methodology is demonstrated by designing tracking fuzzy controllers for vehicle lateral dynamics and water tank systems.

Keywords: Adaptive fuzzy control, Distending function, Membership function

Introduction

Fuzzy theory has been an extensive area of research since its first development, nearly half century ago by Lotfi A. Zadeh [1] and is providing applications in various fields of life [2], [3]. In the field of control system design, complex ill defined non linear processes for which an adequate analytical model is not available, pose a challenging task to meet the control objective using most of the developed linear and non-linear methods. However if operator expertise or knowledge base is available for these systems, fuzzy theory provides an adequate solution for control design [4]. For some non linear process the model parameters vary with the time or with uncertain initial conditions. The control of such non linear dynamic processes falls under the domain of adaptive control, where the control law adapts itself with the changing dynamics to meet the control objective. Adaptive control of aircraft due to changing mass during the flight is most promising application of adaptive control [5]. Design of fuzzy logic control (FLC) is based on the set of 'If then' rules forming a rule base. The Multi input single output fuzzy rule base has the following form

$$\text{if } x_1 \text{ is } A_1^i \text{ And ...... And } x_n \text{ is } A_n^i \text{ Then } y \text{ is } B^i \tag{1}$$

Where $x_1, x_2, ..., x_n, y$ are the linguistic variables which takes the linguistic values from the fuzzy sets $A_1, A_2, ..., A_n, B$ and $i = 1, ..., l$ is the number of fuzzy rule. The Fuzzy Rule Inference (FRI) employs fuzzy relation to represent the rule base as given in 1 for mapping the input and output space. Based on FRI, various type of control schemes have been developed but two most famous are Mamdani [6] and model based Takagi Sugeno (TS) [7] type fuzzy control systems. Mamdani (conventional) type also called Type-I fuzzy control systems, the output of each rule is a fuzzy set. This approach is intuitive and well suited to direct human input and has been shown to meet various control objective successfully [8],[9]. In model based TS (Type-III) fuzzy control the consequent part of the rule base (eq.1) is not a fuzzy set but a crisp value or linear combination of input values. This methods presents fuzzy system model to similar classical control model so the conventional control theory of stability analysis and robustness can be utilized to solve complex control problems. Here we present a new type of fuzzy control design approach. It has the following features:

- It has the advantages of both the above mentioned methods. Both the antecedent and consequent parts of the rule base are fuzzy sets, so its intuitive and close to human inputs.
- Fuzzy arithmetic has been employed to bypass the implication and defuzzification steps and to achieve design simplicity and computation efficiency.

- Here we have introduced a new type of membership function called Distending Membership Function (DMF). Its parameters have semantic meanings. Its so flexible that every type of existing membership function can be approximated by tunning the parameters. It has two variants; Symmetric and Asymmetric distending functions and both can be utilized for control system design.

- Our approach can be modeled as dynamic fuzzy model just like TS model. So the conventional model based control theory can be applied for stability analysis and robustness issues.

Distending Function

Distending function denoted by $\left(\delta_{\varepsilon,\nu}^{(\lambda)}(0=x)\right)$ is a continuous soft equality function which is monotonically increasing in the interval $(-\infty,\ 0)$ and monotonically decreasing in the interval $(0,\ +\infty)$ and takes the real values in $[0,\ 1]$. There are two types distending functions; symmetric and asymmetric.

Symmetric Distending Function

The Symmetric distending function shown in fig. 1 is given by

$$\left(\delta_{\varepsilon,\nu}^{(\lambda)}(0=x)\right) = \delta_s(x) = \frac{1}{1+\frac{1-\nu}{\nu}\left|\frac{x}{\varepsilon}\right|^{\lambda}} \tag{2}$$

where λ is called the sharpness factor, ν is the threshold value and ε is the tolerance factor. $\delta_s(x) : \mathbb{R} \to [0,1]$ $\nu, \varepsilon \in (0,1)$ and $\lambda \in (1,+\infty)$. Also $\delta_s(0=0) = 1$ and $\delta_s(0=\pm\varepsilon) = \nu$.
The distending function changes its shape from trapezoid to a almost a straight line by changing its parameters as shown in the fig. 1

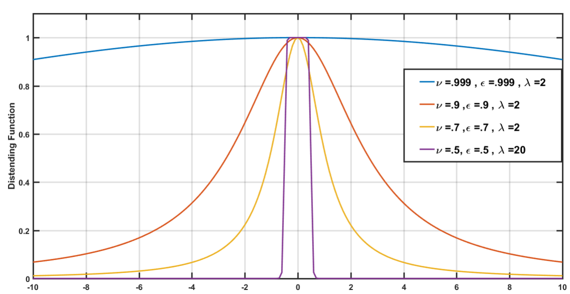

Figure 1: Various shapes of Distending Membership Function

Asymmetric Distending Function

Asymmetric distending function is described by

$$\delta_{\varepsilon_L,\varepsilon_R,\nu_R,\nu_L}^{(\lambda,\lambda_L,\lambda_R)}(x) = \delta_A(x) = \frac{1}{1+\frac{1-\nu_R}{\nu_R}\left(\frac{x}{\varepsilon_R\left(1+e^{-\lambda^* x}\right)}\right)^{\lambda_R}+\frac{1-\nu_L}{\nu_L}\left(\frac{x}{\varepsilon_L\left(1+e^{\lambda^* x}\right)}\right)^{\lambda_L}} \tag{3}$$

where $\delta_A(x) : \mathbb{R} \to [0,1]$ $\nu_R, \nu_L, \varepsilon_R, \varepsilon_L \in (0,1)$ and $\lambda \in (1,+\infty)$. ν_L, ε_L and λ_L are the parameters for the left side whereas ν_R, ε_R and λ_R are the parameters of right side of asymmetric distending function. Whereas λ controls the overall sharpness of the function and usually its fixed. Asymmetric distending function provides more flexibility in the control design problems as right and left side of asymmetric distending function can be controlled independently and can take different shapes depending on the values of parameters similar to the case of symmetric distending function.

Problem description and proposed design approach

Our design methodology is motivated by work in [10] where a fuzzy control design is approached using the concepts of fuzzy arithmetic. From the operator knowledge base or process data the following rule base can be constructed for a multi-input multi-output system (MIMO)

$$\text{if } x_1 \text{ is } A_1^i \text{ And ...... And } x_n \text{ is } A_n^i \text{ Then } y_1 \text{ is } B_1^i \text{ ; ...... ; } y_m \text{ is } B_m^i \tag{4}$$

Where $x_1, x_2, ..., x_n$ are the input linguistic variables which takes the linguistic values from the input fuzzy subsets $A_1, A_2, ..., A_n$ of universe of discourse $X_1, ..., X_n$. $y_1, y_2, ..., y_m$ are the output linguistic variables which takes the linguistic values from the out fuzzy subsets $B_1, B_2, ..., B_m$ of universe discourse Y. $i = 1, ..., l$ are the number of fuzzy rule and m is the number of the outputs of the system. If the output variables are independent of each other then each rule of the rule base given by eq4 can be written as combination of m multi input single output (MISO) rules of the form

$$\text{if } x_1 \text{ is } A_1^i \text{ And ...... And } x_n \text{ is } A_n^i \text{ Then } y_s \text{ is } B_s^i \tag{5}$$

where $s = 1, ..., m$ are the output of the system. For the sake of simplicity we will consider $s = 1$ and will try to find the fuzzy inference mechanism for mapping the input and output space and generating a crisp output for control signal generation as per rule base. However as stated above it can be generalized to MIMO system if outputs are independent of each other.The antecedent part of the each rule is described by a fuzzy relationship function R_i, given by

$$R_i = A_1^i \cap A_2^i \cap ..... \cap A_n^i \tag{6}$$

where the fuzzy operator $\cap$ is given by the general class of fuzzy operators [11]

$$D_\gamma(x) = \frac{1}{1 + \left(\frac{1}{\gamma}\left(\prod_{i=1}^{n}\left(1 + \gamma\left(\frac{1-x_i}{x_i}\right)^\alpha\right) - 1\right)\right)^{\frac{1}{\alpha}}} \tag{7}$$

Almost all the existing conjunctive or disjunctive operators (e.g min/max, product, Einstein, Hamacher, dombi) can be obtained from the generalized operator [11] in eq.7. R_i is defined over the Cartesian space $X_1 \times X_2 \times ... \times X_n$. Let the firing strength of ith rule is w_i, i.e.

$$w_i = w_{i1} \cap w_{i2} \cap ..... \cap w_{in} \text{Where } w_{i1} = Poss[A_1^i \mid x_1] \tag{8}$$

Let $W = \sum_{i=1}^{l} w_i$, then the normalized firing strength of ith rule is define as

$$w_{iN} = \frac{w_i}{W} \text{ and } \sum_{i=1}^{l} w_{iN} = 1 \tag{9}$$

By utilizing the concepts of fuzzy arithmetic [10] and the results of previous section that the linear combination of distending functions is also a distending function, we can state the parameters of the resultant fuzzy output distending function as weighted linear combination of the parameters of l output fuzzy sets . The resultant output distending function has the following form

$$\delta_R(x) = \frac{1}{1 + \frac{1-\nu_R}{\nu_R}\left|\frac{x}{\varepsilon_R}\right|^{\lambda_R}} \qquad \text{and } \nu_R = \sum_{i=1}^{l}(\nu_i w_{iN}),\ \lambda_R = \sum_{i=1}^{l}(\lambda_i w_{iN}),\ \varepsilon_R = \sum_{i=1}^{l}(\varepsilon_i w_{iN}) \quad (10)$$

where ν_i, λ_i and ε_i are the parameters of the ith output fuzzy set. Finally the crisp control output will be the mean coordinate of the resultant distending membership function $\delta_R(x)$.

Simulation and Results

A fuzzy controller is designed using the proposed approach to control the level of the tank at the specified height (reference) by opening/closing of the inlet valve. The difference in the measured level l and reference height called error signal is fed to the fuzzy controller. For controlling the level efficiently, rate of change of the level in the tank is also given to the controller as second input. The controller generates control signal for opening and closing of the valve. The input are fuzzified using the distending membership function. Distending membership functions are also defined for the output control i.e. valve opening/closing signal. Dombi conjunctive and disjunctive operators are selected for evaluating the rules. A reference signal for continuously changing the level of water in the tank between 5m and 15m is fed to the system. The fig.2 shows the comparison of response of proposed control vs the response of the conventional type-I TSK based controller.

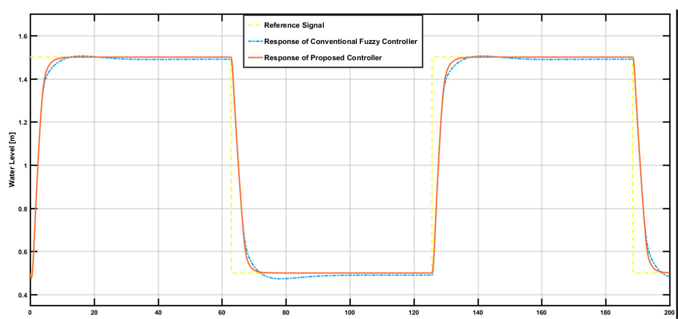

Figure 2: Response Comparison of Proposed and Conventional Fuzzy Controllers

Conclusion and future directions

Fuzzy control based on distending membership function has been proposed. It uses the knowledge rule bases for representing both antecedent and consequent parts and generates the fuzzy model for dynamic processes. Using this model the stability analysis and control design can be achieved. A computationally efficient algorithm has been proposed to design fuzzy controller for dynamic processes. The efficiency of proposed approach has been proved using simulation case study of water tank system . In future, the same design can be extended to design an adaptive fuzzy controller for tunning the membership parameters to overcome the changing process dynamics with increased computational efficiency.

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