

EMPIRICAL VALIDATION OF A DEVELOPMENTAL MODEL OF ARITHMETIC CONCEPTS

S-1

Fritz, Annemarie *, Ehlert, Antje **

** University of Duisburg-Essen, Department of Educational Sciences; University of Johannesburg, Centre for Educational Practice Research*

*** University of Potsdam, Faculty of Human Sciences; University of Johannesburg, Centre for Educational Practice Research*

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Aims: While a growing body of research has revealed strong, individual predictors for mathematical learning, there is still a lack for a structured and comprehensive theory of early arithmetic concepts. In this paper we present the validation of a five-level developmental model of arithmetic concepts: a) a longitudinal study and b) a cross-cultural study with four South African languages.

Theoretical framework: The empirically validated cognitive-psychological orientated model (Ricken, Fritz & Balzer, 2013; Fritz, Ehlert & Leutner, 2018) describes the successive acquisition of five precise arithmetic competencies in the sense of conceptual knowledge for preschool age and grades 1 and 2.

Level 1 (Counting): Gradually, number by number, children start combining the number words with a numerical meaning.

Level II (Ordinal Number Line): Children begin to represent the number word sequence in a kind of mental number line.

Level III (Cardinality): Children learn that a number represents a set and therefore can be decomposed in two or more parts (subsets).

Level IV (Part-Part-Relations): Children deepen their knowledge about the relationship between the whole set and its subsets.

Level V (Equidistant number line intervals): Children start realising that the magnitude of the difference between numbers is always the same (+1).

Method: Study 1: Longitudinal validation. A total of $N=26$ children (17 girls, $M_{age}=71.5$ months, $SD_{age}=10.4$ months) were followed for one and a half years with four measurement times. Each time children's arithmetical conceptual knowledge was assessed with MARKO-D (Ricken et al., 2013), a Rasch-scaled individual test based on the developmental model presented above, which allows to assign children to a specific conceptual level.

Results: In a non-parametrical Page's trend test, person parameters ($L=744.12$, $\chi^2_{(1)}=968.95$, $p<.001$, $\rho=0.724$) as well as conceptual level ($L=728.52$, $\chi^2_{(1)}=970.61$, $p<.001$, $\rho=0.604$) show significant monotonous progress (Page, 1963).

Study 2: The development of arithmetic concepts in South African first-graders.

The second study shows the validation of the German MARKO-D for the use in South Africa in four different languages: English, isiZulu, Afrikaans and SeSotho. After different pilot studies with a total of 2000 children, we ran a norming study with 602 children (322 female; $M_{age}=77.5$ months, $SD_{age}=6.4$ months).

Results: The items of the one-dimensional dichotomous Rasch model showed satisfactory values (weighted infit MNSQ 1 ± 0.2 for 45 out of 48 items; weighted infit MNSQ 1 ± 0.3 for 3 items; person reliability .90). The items form a one-dimensional cumulative scale with five distinguishable segments, according to the levels of the theoretical model.

Discussion: Based on the results we argue that the model is valid and describes children's learning trajectories in arithmetic concepts. The model is thus appropriate to structure teaching and assessment.