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Title: Semi-resolving sets for $\mathrm{PG}(2,q)$

Summary:

Let $G=(A,B;E)$ be a bipartite graph. A subset $S=\{s_1,\dots,s_k\}\subset A$ is a *semi-resolving set for* G, if the ordered distance lists $(d(b,s_1),\dots,d(b,s_k))$ are different for all $b\in B$.

In the talk we consider semi-resolving sets for the incidence graphs of Desarguesian projective planes. In this setting, a semi-resolving set is a point-set $\mathcal{S}$ such that every line has a unique intersection with $\mathcal{S}$. Let $\mu_S(\mathrm{PG}(2,q))$ denote the size of the smallest semi-resolving set in $\mathrm{PG}(2,q)$, and let $\tau_2(q)$ be the size of the smallest double blocking set in $\mathrm{PG}(2,q)$.

We show that $\mu_S(\mathrm{PG}(2,q))\le\tau_2(q)-1$, and if there is a double blocking set of size $\tau_2(q)$ that is the union of two disjoint blocking sets (e.g., if $q\ge 9$ is a square), then $\mu_S(\mathrm{PG}(2,q))\le\tau_2(q)-2$. In the talk we prove the following.

Theorem. *Let $\mathcal{S}$ be a semi-resolving set for* $\mathrm{PG}(2,q)$*, $q\ge 4$. If $|\mathcal{S}|<9q/4-3$, then one can add at most two points to $\mathcal{S}$ to obtain a double blocking set; thus $|\mathcal{S}|\ge\tau_2(q)-2$.*

As a corollary we obtain a lower bound on the size of a blocking semioval. In the proof we use Rédei polynomials and the Szőnyi–Weiner Lemma.