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Title: Projective realization of finite groups

Summary:

Let G be a (finite) group and $\mathcal{P}$ the set of points of a projective plane over the field K. Assume that $\mathrm{char}(K) > |G|$. We say that the disjoint subsets $\Lambda_1, \Lambda_2, \Lambda_3$ of $\mathcal{P}$ *realize* G if there are bijections $\alpha_i : G \to \Lambda_i$ such that for all $g_1, g_2, g_3 \in G$, the points $\alpha_1(g_1)$, $\alpha_2(g_2)$, $\alpha_3(g_3)$ are collinear if and only if $g_1 g_2 = g_3$. The triple $(\Lambda_1, \Lambda_2, \Lambda_3)$ is said to form a *dual* 3*-net* with *fibers* Λ_i.

We describe some constructions: triangular, conic-line type, algebraic, and tetrahedron type. All but the last one are contained in a (possible reducible) cubic curve. The main result is the following.

Theorem (Korchmáros, Nagy, Pace 2012)**.** *Let* $(\Lambda_1, \Lambda_2, \Lambda_3)$ *be a dual* 3*-net of order* $n \geq 4$ *in the projective plane* $PG(2, K)$ *which realizes a group* G*. Then one of the following holds.*

- *(I)* G *is either cyclic or the direct product of two cyclic groups, and* $(\Lambda_1, \Lambda_2, \Lambda_3)$ *is algebraic.*
- *(II)* G *is dihedral and* $(\Lambda_1, \Lambda_2, \Lambda_3)$ *is of tetrahedron type.*
- *(III)* G *is the quaternion group of order* 8*.*
- *(IV)* G *has order* 12 *and is isomorphic to* Alt_4*.*
- *(V)* G *has order* 24 *and is isomorphic to* Sym_4*.*
- *(VI)* G *has order* 60 *and is isomorphic to* Alt_5*.*

Computer calculations show that Alt_4 has no projective realization. This implies that the cases (IV)-(VI) cannot actually occur.

In my talk I focus on one important aspect of the proof, namely, on the situation when the two dual 3-nets of algebraic type share a fiber.