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Title: Hughes planes and their collineation groups

Summary:

We describe the construction of the Hughes plane π based on a nearfield R of order q^2 , with q an odd prime power, whose centre is isomorphic to the finite field $\text{GF}(q)$. Then, we show that the full collineation group of π , say Σ , can be obtained by extending to π the action of all collineations of its Desarguesian subplane π_0 , and taking into account the other collineations induced on π by the automorphisms of the nearfield R . That is, $\Sigma = GK$ with $G = \text{P}\Gamma\text{L}(3, q)$ and $K = \text{Aut}(R)$. One has $\Sigma = G \times K$ if and only if q is a prime, and in this case each collineation of G commutes with each collineation of K . When $q^2 = 9$, $|\Sigma| = 5616 \cdot 6 = 33,696$, whereas for $q^2 = p^{2m} \neq 9$, $|\Sigma| = 2mq^3(q^2+q+1)(q-1)^2(q+1)$. Hence, $|\Sigma| = 2 |\text{P}\Gamma\text{L}(3, q)|$; in particular, if $q^2 = 25$, then $|\Sigma| = 2 \cdot 31 \cdot 30 \cdot 25 \cdot 16 = 744,000$. Finally, Σ has two orbits on π ; namely, π_0 and $\pi \setminus \pi_0$.