

Risk measures in second dual spaces

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In this article we study risk measures in second dual spaces E^{**} where E is a Banach space. For example the space of bounded real sequences ℓ_∞ is the second dual of C_0 , but the class of second dual spaces is very large. The space ℓ_∞ is ordered by the pointwise ordering, or equivalently by the cone ℓ_{∞^+} of positive, bounded real sequences. The constant sequence 1 is considered as the safe asset and the property of monotonicity is taken with respect to the point wise ordering or equivalently by the positive cone ℓ_{∞^+} of ℓ_∞ . A coherent risk measure is a function $\rho: \ell_\infty \rightarrow \mathbb{R}$ so that

- (i) $y \geq x$ in terms of the partial ordering of ℓ_{∞^+} implies $\rho(y) \leq \rho(x)$ (Monotonicity),
- (ii) $\rho(x + a1) = \rho(x) - a, a \in \mathbb{R}$ (Translation Invariance),
- (iii) $\rho(x + y) \leq \rho(x) + \rho(y)$ (Subadditivity),
- (iii) $\rho(\lambda x) = \lambda \rho(x), \lambda \in \mathbb{R}_+$ (Positive Homogeneity).

Note that 1 is an interior point of the ordering cone ℓ_{∞^+} of ℓ_∞ and also that 1 does not belong to the image of the initial space C_0 in it's second dual ℓ_∞ (this image here is again the space C_0).

In this context, we suppose in this article that the set of financial positions as the second dual E^{**} of a Banach space E and we consider coherent risk measures following the usual terminology of the theory of risk measures (see in Artzner et al. (1999), Delbaen (2002), Follmer and Schied (2001)) expanded in general Banach spaces. So we suppose that E^{**} is ordered by a cone P and the safe asset is an interior point of P which does not belong to E . We show that $\rho(x) \geq 0$ for any $x \in E$, therefore any position (vector) of the initial space is not strictly safe. (A position x is strictly safe if $\rho(x) < 0$). This result has some important implications, one of which is a characterization of reflexive spaces in terms of coherent risk measures.

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