34.
ACTIVATION OF METROLOGICALLY USEFUL GENUINE MULTIPARTITE ENTANGLEMENT

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We live in the era of the second quantum revolution [1], where one tries to engineer quantum systems to one's advantage, that is, to outperform what is achievable within the framework of classical physics. In other words, one tries to make technologies out of quantum phenomena. Nowadays, the most prominent branches of quantum technologies are, for instance, quantum computing [2], quantum communication [3], and quantum metrology [4, 5]. In quantum metrology, the main goal is to measure a small phase parameter, θ, with the highest possible precision. For instance, this θ could be the phase difference between the two arms of a Mach-Zehnder interferometer. The more precisely θ can be measured, the more accurately the associated physical quantity (e.g., magnetic field strength, temperature, or time) can be determined. The typical process in quantum metrology is as follows. An initial quantum state ϱ is prepared in an experiment, which is then evolved in time by the system's Hamiltonian operator H in such a way, that the phase parameter θ becomes encoded into the output state $\varrho(H,\theta)$. Subsequently, the output state is measured, and the small parameter θ is estimated via the measurement results. The aim is to achieve the smallest possible uncertainty $\Delta\theta$ of the phase parameter. This process is illustrated in Figure 1.

The error (variance) of this estimation cannot be smaller than the inverse of the so-called quantum Fisher information $F_Q[\varrho, H]$, that is, we have that

$$(\Delta\theta)^2 \geq \frac{1}{F_Q[\varrho, H]},$$

where ϱ is the initial state and H is the system's Hamiltonian operator. It is evident that the larger the value of F_Q, the smaller the error that can be achieved, and thus, the better the metrological performance of the state.

It is known [4,5] that if the initial N-partite state ϱ is not entangled, then $F_Q \approx$ N provides the highest achievable accuracy, which is the so-called 'shot-noise' limit, corresponding to $(\Delta\theta)^2 \approx 1/N$. This is the highest accuracy achievable within the framework of classical physics. When the initial state ϱ is entangled, then $F_Q \approx N^2$ can be achieved, leading to $(\Delta\theta)^2 \geq 1/N^2$, which is a significant improvement when the number of particles N is large. This is the so-called Heisenberg limit, which is the best accuracy achievable within the framework of quantum mechanics.

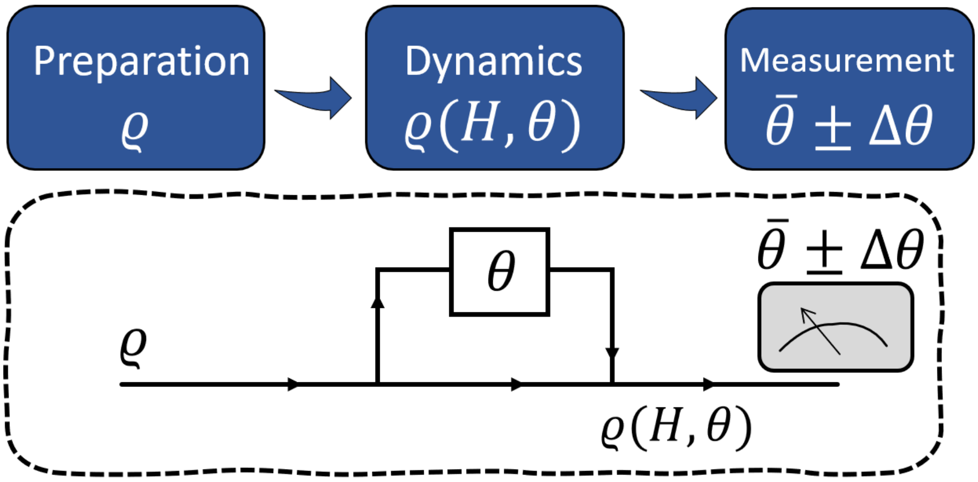

Figure 1: The general process of quantum metrology. The Mach-Zehnder interferometer (below) as a specific implementation, where the goal is to measure the phase difference θ between its two arms. The estimated value of the θ parameter is $\bar{\theta}$, and its standard deviation (error) is $\Delta\theta$.

If an initial state ϱ can achieve an accuracy better than the 'shot-noise' limit with a certain Hamiltonian operator, then the state is considered metrologically useful. In such cases, we have quantum advantage for metrology. That is, quantum mechanical effects are exploited to achieve higher accuracy than what is possible with any conventional, classical (separable) system, where quantum mechanical effects (entanglement) do not play a role. This means that a quantum state is metrologically useful if it has a higher quantum Fisher information value than any separable state. The maximal usefulness can only be achieved by quantum states containing the highest form of entanglement, which is called genuine multipartite entanglement (GME).

It is known that entanglement is a necessary condition for metrological usefulness, but not all entangled states are useful for metrology [6]. In [7], we introduced a simple method to enhance the metrological performance of these non-useful noisy entangled states by utilizing multiple copies of the state. Or in other words, to "activate" their potential to be useful for some metrological task. This is important since the optimal states for metrology (strongly entangled pure states) are very fragile, even small noise can result in the loss of their metrological usefulness. This can be problematic as usually the preparation of the state is not perfect.

With our proposed scheme, we can identify a broad class of practically important states that possess metrologically useful GME in the case of several copies, even though in the single copy case these states can be non-useful, i.e., not more useful than separable states. Thus, we essentially activate quantum metrologically useful GME. Moreover, the maximal metrological usefulness is reached exponentially fast with the number of copies and the necessary measurements are just simple correlation observables. We also provide examples of states not living in the above mentioned class that improve their usefulness (such as noisy GHZ and W states). Our scheme can also be used to protect certain quantum states against certain types of errors without the use of full-fledged quantum error correction techniques.

References
[1] J. P. Dowling and G. J. Milburn, Philos. Transact. A Math. Phys. Eng. Sci. **361**, 1655–1674 (2003).
[2] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O’Brien, Nature **464**, 45–53 (2010).
[3] F. Xu, X. Ma, Q. Zhang, H.-K. Lo, and J.-W. Pan, Rev. Mod. Phys. **92**, 025002 (2020).
[4] G. Tóth and I. Apellaniz, J. Phys. A-Math. **47**, 424006 (2014).
[5] L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Rev. Mod. Phys. **90,** 035005 (2018).
[6] P. Hyllus, O. Gühne, and A. Smerzi, Phys. Rev. A **82**, 012337 (2010).
[7] R. Trényi, Á. Lukács, P. Horodecki, R. Horodecki, T. Vértesi, and G. Tóth, New J. Phys. **26,** 023034 (2024).