On weak convergence of the empirical process with random sample size (Correction)

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Professor P. RÉVÉSZ has pointed out an oversight in the tightness part of the proof of my Theorem 1 [1]; after having applied Doob's identity to the Wiener process there, no m should appear in the integral form, which renders the sum of (1.4) to diverge. The same type of argument works out however by using a recent and very important result of J. KOMLÓS, P. MAJOR and G. TUSNÁDY [3], instead of Kiefer's, which states:

Theorem A [(3]). *On a rich enough probability space one can define positive absolute constants A, B, C and, for each n, a Brownian Bridge $\{B_n(x);\ 0 \leqq x \leqq 1\}$ such that*

$$P\{\sup_{0\leqq x\leqq 1} \sqrt{n}\,|Y_n(x)-B_n(x)| \geqq A\log n+z\} \leqq Be^{-Cz}$$

for all real z, where $Y_n(x)=\sqrt{n}\,(F_n(x)-x)$ is the empirical process.

From now on we assume that the probability space $\{\Omega, \mathscr{B}, P\}$ of the Introduction of [1] is already that of Theorem A here, on which we also assume that a standard Wiener process $\{W(t);\ 0\leqq t<\infty\}$ is also defined.

Going back to the left hand side of (13) in [1], which is bounded above by

$$0+\lim_{\delta\to 0}\overline{\lim_{n\to\infty}}\, P\{\max_{n(a-\varrho)\leqq m\leqq n(b+\varrho)}\ \sup_{|s-t|<\delta} |Y_m(s)-Y_m(t)| > \varepsilon\},$$

the probability herewith can, in turn, be majorized by

$$2\sum_{m=[n(a-\varrho)]}^{[n(b+\varrho)]} P\left\{\sup_{0\leqq s\leqq 1}\left|Y_m(s)-\frac{W(ms)-sW(m)}{\sqrt{m}}\right| \geqq \frac{\varepsilon}{4}\right\}+ \tag{14}$$

$$+P\left\{\max_{n(a-\varrho)\leqq m\leqq n(b+\varrho)}\ \sup_{|s-t|<\delta}\left|\frac{W(ms)-sW(m)}{\sqrt{m}}-\frac{W(mt)-tW(m)}{\sqrt{m}}\right| \leqq \frac{\varepsilon}{2}\right\}.$$

The term in the second row of (14) is $\leqq$

$$P\left\{\max_{n(a-\varrho)\leqq m\leqq n(b+\varrho)}\ \sup_{|s-t|<\delta}\left|\frac{W(ms)}{\sqrt{m}}-\frac{W(mt)}{\sqrt{m}}\right|\geqq\frac{\varepsilon}{4}\right\}+$$

$$+P\left\{\max_{n(a-\varrho)\leqq m\leqq n(b+\varrho)}\frac{\delta}{\sqrt{m}}|W(m)|\geqq\frac{\varepsilon}{4}\right\}.$$

Now the $\lim\limits_{\delta\to 0}\overline{\lim\limits_{n\to\infty}}$ of the first term of this sum is seen to be zero via Lemma of [2], while that of the second via Kolmogorov's inequality. As to the sum in the first row of (14) we apply Theorem A with $z=\frac{\varepsilon}{8}\sqrt{m}$ and, replacing $\frac{W(ms)-sW(m)}{\sqrt{m}}$ by another Brownian Bridge if necessary, we get, for n large enough, the upper bound

$$2\sum_{m=[n(a-\varrho)]}^{[n(b+\varrho)]} Be^{-C\frac{\varepsilon}{8}\sqrt{m}},$$

and also zero in the limit.

References

[1] S. Csörgő, On weak convergence of the empirical process with random sample size, *Acta Sci. Math.*, **36** (1974), 17—25.

[2] M. Csörgő and S. Csörgő, On weak convergence of randomly selected partial sums, *Acta Sci. Math.*, **34** (1973), 53—60.

[3] J. Komlós, P. Major and G. Tusnády, An approximation of partial sums of independent RV's and the sample DF. I, Preprint No. 72/1974 of Math. Inst. Hung. Acad. Sci. To appar in *Z. Wahrscheinlichkeitstheorie verw. Geb.*

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