

Bibliographie

Iain T. Adamson, Elementary Rings and Modules, (University Mathematical Texts) 136 pages, Edinburgh, Oliver and Boyd, 1972.

This book is an elementary introduction to some basic notions and ideas of commutative algebra. It is well-written and self-contained in the sense that it contains an explanation of the steadily used general algebraic concepts. Thus it is very suitable for undergraduate students as a textbook. Examples and exercises, a reading list and an index complete this small but useful book.

Béla Csákány (Szeged)

F. F. Bonsall and J. Duncan, Complete Normed Algebras (Ergebnisse der Mathematik und ihrer Grenzgebiete, 80), X+301 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1973.

The theory of Banach algebras has wide-ranging application in harmonic analysis, operator theory and function algebras. Moreover it disposes of a rich collection of general results some of which deserve to be known by every mathematician.

The present book is an excellent account of the principal methods and results in the theory of Banach algebras, both commutative and noncommutative. It is a new and indispensable source for anyone, student or researcher, working in this area. The highly developed theory of C^* -algebras, function algebras and group algebras is outside the scope of this monograph as well as the multipliers, the extensions and other generalizations of Banach algebras.

Chapter 1: Concepts and Elementary Results. It deals with elementary facts on normed algebras, inverses, equivalent norms, the spectrum and contour integrals. An elegant exposition is given for the functional calculus, the elementary functions, the numerical range and the approximate identities. Normed division algebras conclude the chapter.

Chapter 2: Commutativity. It begins with the characterization of multiplicative linear functionals and the Gelfand representation theory of commutative Banach algebras. A concise and elegant account of derivations, joint spectra and the functional calculus for several elements succeeds. The final topic of this chapter is the theory of functions analytic on a neighbourhood of the carrier space, the Shilov boundary and the hull-kernel topology of that space.

Chapter 3: Representation Theory. After some algebraic preliminaries the continuity of the irreducible representations of a Banach algebra on normed spaces is discussed. Also, the structure space of a noncommutative algebra, the A -Module pairings, and the dual modules are treated here. The theory of representations of linear functionals concludes the chapter.

Chapter 4: Minimal ideals. The necessary algebraic arrangement is followed by the theory of annihilator algebras. Recent results on compact action on Banach algebras and the basic facts on H^* -algebras are given.

Chapter 5: Star Algebras. The theory initiated by Gelfand and Naimark of representation of star algebras by positive linear functionals is treated. Recent results on characterizations of C^* -algebras and B^* -semi-norms and the new, nice theory of hermitian algebras conclude the chapter.

Chapter 6: Cohomology. Tensor products, amenable Banach algebras and the recent theory of cohomology of Banach algebras are treated.

Chapter 7: Miscellany. It begins with capacity and positiveness of the spectrum in Banach algebras. The theory of type O and locally compact semi-algebras is discussed. Recent results on characterization of Q-algebras conclude the book.

The presentation of the book attains an optimum in consistency and readability. Summarizing, the authors did an important and beautiful job in writing this monograph.

Zoltán Sebestyén (Budapest)

André Delachet, La géométrie élémentaire, Le calcul vectoriel, Le calcul tensoriel, 128, 128, 128 pages, Presses Universitaires de France, Paris, 1966, 1967, 1969.

These little books were published in the series "Que sais-je?". No mathematical knowledge beyond the secondary school level is necessary to read them. They were written mostly for beginners of higher training, but they are also useful and well constructed introductions to these areas of mathematics for everybody interested in mathematics.

"La géométrie élémentaire" treats the axiomatic foundations of the geometry of the plane using the concept of set and real number. It follows the method of M. Choquet and gives a good insight into the axiomatic method.

"Le calcul vectoriel" deals with vectors in 3 dimensional physical space. After the definitions of the concept of vectors and the fundamental operations with them it discusses many physical applications: statics of the rigid solid, speed of the moving point in different systems of co-ordinates, the moving of a rigid solid. It also touches upon the bases of differential geometry.

"Le calcul tensoriel" supposes some knowledge in Linear Algebra. In the first part the tensor algebra (concept of tensor product, affine tensors, exterior algebra, Euclidian tensors) are treated. The second part deals with tensor fields defined in Euclidian and Riemannian spaces and their covariant derivatives.

L. Kérchy (Szeged)

W. W. Comfort, S. Negrepointis, Continuous Pseudometrics, 126 pages, New York, M. Dekker, Inc., 1975.

The book contains a detailed exposition of an interesting part of general topology. The topics concerned belong for a large part to what may be called, in a broad sense, the descriptive theory of sets in topological spaces. The title does not perhaps express the content quite fully, though pseudometrics are used quite often indeed, both explicitly and implicitly.

As the authors point out, the book is based on classroom notes elaborated during the academic year 1967—68. The exposition is clear and presupposes almost no knowledge of general topology.

In Section 1 the general concepts of P-embedding and P-fication are introduced for an arbitrary class P of completely regular spaces (if, e.g., P is the class of compact spaces, then Y is the Stone-Čech compactification of X if and only if Y is a P-fication of X and X is P-embedded in Y). Using these notions, the basic facts concerning paracompact, realcompact, topologically complete, etc., spaces are presented in a systematic and lucid way in Sections 2 through 7. A number of deeper results, e.g. Glicksberg's theorem on the Stone-Čech compactification $\beta(X \times Y)$ are included.

In Section 8 Borel metrisable separable spaces are considered. Sections 9—11 contain some topics that are not quite current and seem to appear in a book for the first time. In Section 10, the local connectedness of βX is examined. In Section 9, the authors consider Baire sets and Baire spaces, defined as follows: X is Baire in $Y \supset X$ if X can be obtained from zero-sets (i.e. sets $f^{-1}0$ where $f: Y \rightarrow \mathbb{R}$ is continuous) in countably many steps by taking countable unions and intersections, X is a Baire space if it is Baire in βX . Section 11 contains counterexamples.

The book is a useful exposition of important and interesting topics including also some less known results. It may well serve as a basis of a special course in general topology, or of a part of a more extensive introductory course.

M. Katětov (Prague)

Steven A. Gaal, Linear Analysis and Representation Theory (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 198), IX+688 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1973.

The book is an introduction to a number of topics in functional analysis, harmonic analysis and representation theory in Hilbert space. In the Preface the author says: "... I tried not to be encyclopedic but rather select only those parts of each chosen topic which I could present clearly and accurately in a formulation which is likely to last. The material I chose is all mathematics which is interesting and important both for the mathematician and to a large extent also for the mathematical physicist". The book is intended for "... frequent browsing, consultation and other occasional use". The exposition is clear and concise and proofs are as simple in concept as possible, so Gaal's work can serve as a textbook for students and also as a reference book. The chapters are made "as independently readable as possible under the given conditions". The chapter headings will give the reader of this review some idea of the book's content: Algebras and Banach Algebras; Operators and Operator Algebras; The Spectral Theorem, Stable Subspaces and v . Neumann Algebras; Elementary Representation Theory in Hilbert Space; Topological Groups, Invariant Measures, Convolutions and Representations; Induced Representations; Square Integrable Representations, Spherical Functions and Trace Formulas; Lie Algebras, Manifolds and Lie Groups. A bibliography, subject index, and index of notations and special symbols are provided

J. Szűcs (Szeged)

I. C. Gohberg—M. G. Krein, Introduction to the Theory of Linear Nonselfadjoint Operators: (Translations of Mathematical Monographs, Vol. 18), XV+378 pages, American Mathematical Society, Providence, R. I., 1969.

During the last 20—25 years the theory of non-selfadjoint operators in Hilbert space grew to an important branch of functional analysis, which has now its own methods and typical results. In the Soviet Union research in this direction was started by M. V. Keldyš, M. S. Livšic and L. A. Sahnovi, and intensively and on a very wide scale continued by M. G. Krein, M. S. Brodskii, I. C. Gohberg, V. I. Macajev and others. Their investigations can be characterized by intense use of results of complex function theory in connection with estimates of the resolvent of a given operator, with (infinite) perturbation determinants and characteristic functions. An essential role in these investigations is played by ideals of compact operators, typical results are e.g. statements about the completeness of the system of root vectors of an operator or an operator pencil, canonical representations of the operator (which can be considered as generalizations of the triangular representation of a quadratic matrix) and abstract factorizations.

This monograph (the original edition in Russian appeared in 1965) contains the foundations of this theory and its results about the mentioned completeness problems. After a first introductory

chapter (here one finds e.g. Gohberg's theorem on holomorphic operator functions) the second chapter contains a systematic exposition of the theory of s -numbers (absolute eigenvalues) of a compact operator (e.g. the inequalities of H. Weyl, Ky Fan, A. Horn). They form a basis for a systematic study of the symmetrically-normed ideals of the ring of bounded linear operators in Hilbert space in the third chapter, where also — compared with the classical exposition by R. Schatten — many new results can be found. In the fourth chapter the theory of perturbation determinants and some of its applications are given. Especially important are here the connections of the hermitean components of a Volterra operator. These and other methods are used in the fifth chapter in order to prove deep results on the completeness of root vectors of certain classes of operators. Finally, in the last chapter some notions of a basis in a Hilbert space are discussed and bases of root vectors of certain dissipative operators are considered. Most of the material of the book is here for the first time exposed in a monograph, many of the proofs had not at all been published before. So the appearance of this book was one of the highlights in the theory of non-selfadjoint operators. During the past ten years it became a frequently quoted standard reference with great influence on the development of operator theory. In a certain sense it has a continuation in the book *Theory of Volterra operators in Hilbert space and its applications* by the same authors, where e.g. questions of canonical representations and factorizations are treated. These two books and the monograph by Sz.-Nagy and C. Foiaş: *Harmonic analysis of operators on Hilbert space* cover a great and essential part of what is nowadays known about non-selfadjoint operators on Hilbert space.

H. Langer (Dresden)

Nathan Jacobson, Basic Algebra. I, XVI+472 pages, San Francisco, W. H. Freeman and Co., 1974.

This book is a very attractive introduction to the abstract algebra for undergraduate students; it is also an excellent tool to refresh and supplement the knowledge of their teachers. The author of *Lectures in Abstract Algebra* hardly needs any praise for conciseness and clarity, but it is impossible to leave unmentioned his excellent ability, amply proved again by this book, to present algebra in a modern and gently simple style. This does not mean that there are merely easy facts dealt with here; e.g. the transcendence of π found room in the book as well as the description of several sequences of finite simple groups.

The volume consists of eight chapters (preceded by an introduction where some basic set-theoretic and arithmetic facts are summarized). To make feel its flavour, let us list them here: 1. Monoids and groups. 2. Rings. 3. Modules over a principal ideal domain. 4. Galois theory of equations. 5. Real polynomial equations and inequalities. 6. Metric vector spaces and the classical groups. 7. Algebras over a field. 8. Lattices and Boolean algebras.

Béla Csákány (Szeged)

N. S. Landkof, Foundations of modern potential theory (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 180), X+424 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1972.

This monograph on potential theory is a translation, by A. P. Doohovskoy, of the original in Russian: *Osnovy sovremennoĭ teorii potenciala*, Nauka (Moscow, 1966). At that time there was no monograph on potential theory which presented at a sufficiently modern level the "analytic" part of the theory relating to concrete kernels. This book remedied that deficiency in the literature. Although the book is directed to mathematicians who wish to be introduced to potential theory for the first

time, it will have interest also for specialists. It is important that knowledge of classical potential theory is not required for the reading of this book but the ideas and methods of modern function theory, functional analysis and general topology are necessary. The entire exposition is devoted to M. Riesz kernels and Green kernels. To justify such a selection of kernels the author says in the preface: "First, M. Riesz kernels include, as special (or limiting) cases, the classical Newtonian and logarithmic kernel. Second, changing the character of the singularity of a kernel leads to, from the point of view of analysis, very deep alterations of the theory: this is because the Laplace differential operator has to be replaced by a non-local integro-differential operator. With regard to Green kernels, they are essentially a model with which a potential theory for more general elliptic differential operators can be constructed."

The chapter headings are: Introduction, I. Potentials and their basic properties, II. Capacity and equilibrium measure, III. Sets of capacity zero. Sequences and bounds for potentials, IV. Balayage, Green functions, and the Dirichlet problem, V. Irregular points, IV. Generalisations (Paragrapgs of this chapter: 1. Distributions with finite energy and their potentials, 2. Kernels of more general type, 3. Dirichlet spaces). Almost all references to the literature are at the end of the book under "Comments and Bibliographic References".

The book is well organized, the presentation of the material is concise but understandable, its format is nice.

I. Szalay (Szeged)

A. M. Olevskii, Fourier Series with Respect to General Orthogonal Systems (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 86), VIII+136 pages, Springer Verlag, Berlin—Heidelberg—New York, 1975.

The last decade has been a period of intensive development in the theory of Fourier series. Advances have also been made in the theory of Fourier series with respect to general orthogonal systems. In particular, it was discovered that several results which had been seen to depend on the special peculiarities of the trigonometric system have in fact a considerably more general nature and are determined by such properties of ON systems as completeness or uniform boundedness.

The present book is primarily based on the investigations of the last fifteen years concerning Fourier series with respect to general ON systems. Results involving specific systems are examined only to the extent that they shed light on the problems of the general theory. The author does not touch at all upon the investigation of multiple Fourier series and spectral expansions, or upon multiplicative systems and other special classes of ON systems.

The fundamental results are given with proofs. However, the author has tried to avoid letting the technical details encumber the presentation. A number of the results of Chapters I and III were formerly only announced by the author and are now for the first time set forth in detail.

The main result presented in Chapter I is that there is no uniformly bounded ON system with respect to which every continuous function has an everywhere convergent Fourier series expansion. Thus the phenomenon of the local divergence of a Fourier series, discovered by du Bois—Reymond at the end of the last century, is connected not specifically with the trigonometric system, but has a general nature, it arises with any bounded ON system. A basic consequence is that no uniformly bounded ON system can form a basis in the space C of the continuous functions on a finite interval. Further, the uniform boundedness of an ON system determines a fixed order of growth of the Lebesgue functions $L_n(x)$. Namely, such systems always satisfy the relation $L_n(x) \neq o(\log n)$ on a set of positive measure. The proof of these results and others connected with them are based on a new method of estimating a lower bound, in the metric of the space L^1 , for the partial sums of series of orthonormal functions.

Chapter II deals with conditions on the coefficients that ensure a.e. convergence. The general problem is to describe, for a given ON system φ , the class $\mathfrak{S}(\varphi)$ of sequences $\{c_n\}$ of coefficients for which the series $\sum c_n \varphi_n$ converges a.e.. Only in rare instances is there an effective solution to this problem; one of these exceptions is the Rademacher system r , for which $\mathfrak{S}(r) = l_2$. It is therefore of interest to study the intersection (or the union) of the classes $\mathfrak{S}(\varphi)$ for various sets of ON systems. Special attention has been given to the class $\mathfrak{S}_\Omega = \bigcap_{\varphi \in \Omega} \mathfrak{S}(\varphi)$, where Φ is the set of all ON systems. This chapter presents, among others, Tandori's results based upon the further development of Men'shov's method, Garsia's theorem on that the terms of any orthogonal series from L^2 can be arranged in such an order as to make the series converge a.e., etc.

In the last few years it was discovered that the Haar system $\{\chi_n\}$ plays a specific role in the class of all complete ON systems. A method is presented in Chapter III that permits in a number of cases a reduction of a problem for an arbitrary complete ON system to the same problem for $\{\chi_n\}$. Roughly speaking, if a Fourier series divergence phenomenon occurs with the Haar system, then such a phenomenon is unavoidable for every complete ON system. For example, Ulyanov and the author proved, independently of each other, that for any complete ON system there exists a function from L^2 whose Fourier series after appropriate rearrangement of its terms diverges a.e.; $\{\chi_n\}$ has the smallest possible Banach constant under fairly general conditions on Banach spaces B of functions; either $\{\chi_n\}$ forms an unconditional basis in B or there does not exist any unconditional basis at all in this space. In the rest of this chapter the behaviour of the Fourier coefficients of continuous functions is studied, the local Carleman singularity is extended for any complete ON system and a variety of related results is proved.

Chapter IV is devoted to the a.e. and mean convergence of Fourier series with respect to general ON systems, but in contrast to Chapter II, properties here are first stipulated on the function, and not on the coefficients of the expansion. Considerable attention is paid to the peculiarities of Fourier series in the spaces L^p , $p < 2$. In this case new phenomena arise, which do not in the case of L^2 -series. For example, there exists an ON system, closed in C , whose Lebesgue functions are uniformly bounded, and nevertheless, the Fourier series of some $f \in \bigcap_{p < 2} L^p$ diverges a.e.; Garsia's theorem does not extend to the spaces L^p , $p < 2$; there exists a function in $\bigcap_{p < 2} L^p$ whose Fourier series, with an appropriate ordering, can represent any measurable function, etc.

The book has been carefully and accurately written. The presentation is concise but always clear and well-readable. The main goal of the author is to survey the subject as it exists today, and it is perhaps not exaggerated to assert that this goal has been completely attained. It will certainly indicate the weak and strong spots in the edifice of the theory built so far, and thereby facilitate future research.

F. Móricz (Szeged)

Proceedings of the Symposium on Complex Analysis, Canterbury, 1973 (London Mathematical Society, Lecture Note Series 12), Edited by J. Clunie and W. K. Hayman, VII + 180 pages, Cambridge University Press, 1974.

Part I, containing the contributions, mostly in short abstracts, of the participants, gives an interesting cross-section of some of the domains of present research in Complex Analysis. In the shorter Part II, W. K. Hayman gives a report on the progress on problems stated at a previous conference in the same area, at Imperial College, London, in 1964, and lists new problems that arose from the present symposium.

J. Terjéki (Szeged)

Sergiu Rudeanu, Boolean functions and equations, XIX+442 pages, Amsterdam—London—New York, North-Holland—American Elsevier Publ. Co., 1974.

Since the thirties the theory of Boolean algebras has developed in two very different lines: in a set-theoretical and in an algebraic direction. The set-theoretical approach, which goes back to Stone's representation theory, has become well-known and well-developed during the last decades and now there are three excellent monographs (those of Sikorski, Dwinger and Halmos) concerning this subject.

Curiously enough, the older algebraic approach, the theory of Boolean functions and of solutions of Boolean equations, which was intensively studied by Boole himself, Peirce, Poretski, Schröder, Löwenheim and other outstanding mathematicians of the last century, is much less familiar in the present mathematics. Its development has become scattered in the twentieth century, although it has been investigated by the same intensity as previously. This undesirable situation led to the rediscovery of a number of results and it has become indispensable for the further development to summarize and unify the theorems on this line into a "homogeneous" theory. This is done in the book of professor Rudeanu, one of the eminent specialists in this field.

The book consists of two parts.

Part I is devoted to the abstract theory of Boolean (systems of) equations and includes fundamental theorems concerning among others the solvability of Boolean equations, orthonormal solutions, symmetric equations, Boolean ring equations, Boolean transformations, parametric equations, syllogisms, Boolean arithmetic, Boolean geometry and Boolean calculus.

Part II, which is written in an informal style, deals with applications to switching theory.

Both parts are self-contained. For this reason the second part includes a brief summary of the first one. The book is completed by a bibliography consisting of more than 350 items.

This is a basic book for anybody who studies Boolean equations or would like to understand the mathematical foundations of switching theory.

András P. Huhn (Szeged)

Robert M. Switzer, Algebraic Topology — Homotopy and Homology (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band, 212) XII+526 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1975.

In the last few years several excellent textbooks have appeared on algebraic topology. Of these in the author's opinion the most notable is the *Algebraic topology* of E. H. Spanier. The present book offers more than any of these: it brings the reader to a point from which he can "begin research in certain areas of algebraic topology: stable homotopy theory, K-theory, cobordism theories". Of course it does not try to achieve the same very advanced level in all areas of algebraic topology, the choice is heavily influenced by the author's research interests. Although it goes considerably further than Spanier's book, there is a certain overlap between this and Spanier's book — especially in Chapters 0—6, 14 and 15 — thus Spanier's book is recommended by the author as a companion volume to his one.

The book under review has grown out from courses given by the author at the University of Manchester in 1967—1970, at Cornell University in 1970—1971 and at the Georg August University, Göttingen, in 1971—1972. It assumes the knowledge of the rudiments of algebraic topology, including singular homology, the fundamental group and covering spaces, and Chapter 12 also assumes some familiarity with differentiable manifolds.

The book is divided into 21 chapters. Chapters 0 and 1 contain some results from set-theoretic topology which are repeatedly used in the text and the basic definitions of category theory. Chapter

2 introduces the sets $[X, Y]$ of homotopy classes of maps $f: X \rightarrow Y$ and studies the question of when is $[X, Y]$ a group, when is a sequence $[X, W] \xrightarrow{f_*} [Y, W] \xrightarrow{g_*} [Z, W]$ exact, etc. Chapter 3 specializes to $X = S^n$, the n -sphere and considers $\pi_n(Y, y_0) = [S^n, s_0; Y, y_0]$, the n th homotopy group of Y ($n \geq 1$). The more elementary properties of these groups are proved in this chapter. In Chapter 4 the notions of fibration, weak fibration and fibre bundle are introduced and it is shown that every fibre bundle is a weak fibration. Examples of fibre bundles are given. The homotopy groups of S^n and $T^n = S^1 \times \dots \times S^1$ are computed using the observation that a covering $p: X' \rightarrow X$ is a fibre bundle with discrete fibre. Chapter 5 gives the notion and some straightforward properties of CW-complexes. Chapter 6 contains some finer homotopy results on CW-complexes: $\pi_n(X, x_0)$ depends only on the cells of dimension at most $n+1$; the suspension homomorphism $\Sigma: \pi_q(X, x_0) \rightarrow \pi_{q+1}(SX, *)$ ($[f] \rightarrow [1 \wedge f]$) is an isomorphism for $q < 2n+1$ if X is an n -connected CW-complex, etc.. Chapter 7 turns from homotopy theory to homology and cohomology theories. It introduces the notion of generalized homology theory by means of the first six Eilenberg-Steenrod axioms and studies the direct consequences of these axioms. Chapter 8 shows how to construct homology and cohomology theories. In Chapter 9 it is shown that in Chapter 8 all possible cohomology theories on the category of CW-complexes have been constructed. In Chapters 10, 11 and 12 three important examples are given: ordinary homology, K-theory and bordism. Chapter 13 is devoted to the study of product in homology and cohomology. The next chapter applies products to duality and orientability questions. In Chapter 15 comes the introduction of spectral sequences and the succeeding chapter is concerned with characteristic classes. The headings of the last four chapters read as follows: Cohomology Operations and Homology Cooperations; The Steenrod Algebra and its Dual; the Adams Spectral Sequence and the e -Invariant; Calculation of the Cobordism Group.

The book's bibliography does not pretend to be comprehensive, since this is unnecessary because of the existence of Steenrod's compendium of all mathematical reviews related to topology. Instead, the bibliography has two goals: "(1) to suggest to the student where he might begin to pursue a given topic further and (2) to acknowledge the sources from which much of the material ... is drawn". A subject index is also given.

J. Szűcs (Szeged)

Robin J. Wilson, Introduction to graph theory, VIII+168 pages, Edinburgh, Oliver and Boyd, 1972.

A decade ago there existed 2—3 textbooks on graph theory; this number has grown very rapidly in the last years and books at different levels — introductory and advanced, general and special, "pure" and "applied" — have been written.

This book is intended to serve as "an inexpensive introductory text on the subject, suitable both for mathematicians taking courses in graph theory and also for non-specialists wishing to learn the subject as quickly as possible". Although several other books would meet this program, the present one is certainly one of the most successful attempts inasmuch its relative shortness is coupled with a fortunate selection of non-trivial concepts, results and applications. Wilson manages to avoid the dangers writing about such a broad and widely applicable subject. He not only defines the basic notions and illustrates them by well-chosen examples (this could have resulted in a book showing graph theory as a mere language without own mathematical contents) but also states those basic results which now should belong to the arsenal of anyone wishing to apply graph theory. On the other hand, he does not go into those proofs which are too long or too technical.

The book covers the following topics: Eulerian and Hamiltonian graphs (finite and infinite); trees and their enumeration; planarity and duality (stating Kuratowski's characterization but proving only Whitney's); coloring of graphs and chromatic polynomials; digraphs with applications

to Markov chains; matchings (König-Hall Theorem); network flows and Menger's Theorem; matroids (describing the spectacular results on graphical representation of matroids). There are about 250 exercises, where several other related problems are touched.

Most of the material is presented in a neat, enjoyable way. The introductory chapters, with many well-chosen examples, are particularly well-written. I have found the chapter on Whitney-duality (§ 16) confusing; this notion is equivalent to abstract-duality of the preceding paragraph but this is explicitly mentioned in the proof of Theorem 16.c only. Instead of speaking about 3 kinds of duality, to state Whitney's definition as a characterization of the abstract-dual would have made this chapter much clearer.

Summarizing, this book is a well-written introductory text for those wishing to learn the basics of graph theory in such a way (the only reasonable way, in my opinion) that they also want to get a picture on how proofs, conjectures, notions, applications arise in this field.

László Lovász (Szeged)

T. Yoshizawa, Stability theory by Liapunov's second method (Publications of the Mathematical Society of Japan, Volume 9), VII+223 pages, The Mathematical Society of Japan, 1966.

T. Yoshizawa, Stability theory and the existence of periodic solutions and almost periodic solutions (Applied Mathematical Sciences, Volume 14), VII+233 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1975.

The direct or second method of Liapunov became one of the most important tools of the qualitative theory of ordinary differential equations. It is known that several properties of the motions of conservative mechanical systems follow from the law of the conservation of energy. This observation is generalized in Liapunov's second method: from the properties of a suitable vector-scalar function there follow certain properties of the motions of a general dynamical system without explicit knowledge of the motions. Many extensions, refinements, applications on stability theory, on the asymptotic behaviour and boundedness of solutions, etc., show that the theory built on this method has grown with a splendid speed. This method plays an important role also in the theory of control systems, dynamical systems, and functional-differential equations.

The author reached important results in the development of this method and also in its applications in various problems (e.g. in the characterization of the boundedness of various types by Liapunov functions; in the study of the stability and the boundedness of the solutions of functional-differential equations; in the discussion of the asymptotic behaviour and stability properties for periodic and almost periodic systems). Naturally, these topics of the stability theory play a central role in both monographs. But at the same time the first book and the second chapter of the second book give a good survey on the development and main results of Liapunov's second method.

The second monograph originates from a seminar on stability theory given by the author at the Mathematics Department of Michigan State University during the academic year 1972—73. As an introduction we get a guide for properties of almost periodic functions with parameters as well as for properties of asymptotically almost periodic functions. In the most interesting part of the book the converse theorem on integrally asymptotic stability and the relationship between total stability and other types of stability are treated including the newest results. Then the existence of a periodic solution in a periodic system is discussed in connection with the boundedness of solutions, and the existence of an almost periodic solution in an almost periodic system is considered in connection with some stability property of a bounded solution. Finally, sufficient conditions for the existence of a unique uniformly asymptotically stable periodic (almost periodic) solution of a periodic (almost periodic) system are proved by the aid of Liapunov functions.

The books are very useful and important for everybody interested in the qualitative theory of ordinary differential equations, especially in the applications of Liapunov's second method.

Chapter headings (*first book*): Preliminaries; Liapunov stability and boundedness of solutions; Extensions of stability theory; Extreme stability and stability of sets; Converse theorems on stability and boundedness; Perturbed systems; Existence theorems for periodic solutions and almost periodic solutions; Functional-differential equations; (*second book*): Preliminaries; Stability and boundedness; Existence theorems for periodic solutions and almost periodic solutions.

L. Hatvani—L. Pintér (Szeged)

Helmut H. Schaefer, Banach Lattices and Positive Operators (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 215), XI+367 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1974.

The end of the author's earlier book "Topological Vector Spaces" contains a brief summary of results in topological vector lattices. Since the first edition of that book, the theory of Banach lattices developed into an independent theory, though it has interesting applications to functional analysis, and its results can be applied even in the theory of topological vector spaces. This situation is well reflected in the present book.

The first chapter deals with positive complex matrices to supply a unified discussion of the most important operator theoretic properties of positive matrices and to serve as a motivation for the study of positive operators on Banach lattices. The basic properties of Banach lattices are treated in Chapter II; some special Banach lattices are also considered. The purpose of the next chapter is to look for a representation theory of Banach lattices, similar to that of commutative Banach algebras. A beautiful result is a generalization of the Halmos-von Neumann theorem on ergodic dynamical systems. Each section of the book treats very interesting questions, as tensor products of Banach lattices, lattices of operators between Banach lattices, Hilbert lattices, the peripheral spectrum of positive operators, to mention only some of them.

The style of the author is the same as in his earlier book; the presentation of the material is concise but easily readable, the notations are suggestive and exact. We can surely assert that this book is unique nowadays.

T. Matolcsi (Budapest)

Roger Temam, Analyse numérique (Collection SUP "Le Mathématicien", 3), 119 pages, Paris, Presses Universitaires de France, 1970.

La résolution approchée des équations fonctionnelles constitue une partie importante de l'analyse numérique. Ce livre sert d'une bonne introduction à la théorie des résolutions approchées et il donne aussi des exemples pratiques. La première partie traite de quelques aspects de l'approximation de la solution d'une équation de type elliptique, par exemple le théorème de Lax-Milgram et la méthode de Galerkin, le problème de l'approximation par des éléments non appartenant à l'espace considéré, et l'estimation de l'erreur. Les résultats sont appliqués aux problèmes de Dirichlet et de Neumann.

Le livre est bien construit et on peut le lire aisément. La lecture ne suppose qu'une certaine connaissance des espaces hilbertiens, éléments de la théorie de la mesure et de la théorie des distributions.

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Wolfgang Walter, Einführung in die Theorie der Distributionen, VIII+211 Seiten, Mannheim—Wien—Zürich, Bibliographisches Institut—Wissenschaftsverlag, 1974.

Die Theorie der Distributionen ist von grosser Wichtigkeit sowie in der reinen als auch in der angewandten Mathematik. Heutzutage gibt es viele guten Bücher, die eine Einführung in diese Theorie bieten. Man kann auch diese Buch zu ihnen zählen. Wenn man ein solches Buch in der Hand hält, sucht man, was es von den anderen unterscheidet. Dieses Buch kann leicht und mit Interesse gelesen werden und sein letzter Paragraph hat eine kurze Einführung in die Theorie der Sobolev-Räume zum Inhalt. Leider werden die konventionellen Bezeichnungen, die nicht konsequent und manchmal auch irreführend sind, gebraucht, obwohl man schon ein ausgezeichnetes Bezeichnungssystem für Distributionen vorhanden hat.

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LIVRES REÇUS PAR LA RÉDACTION

- F. W. Anderson—K. R. Fuller, Rings and categories of modules** (Graduate Texts in Mathematics, Vol. 13), IX+339 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1974. — DM 36,30
- A. C. Bajpai—I. M. Calus—J. A. Fairley, Numerical methods for engineers and scientists**. A students' course book, XII+380 pages, London, Taylor and Francis Ltd., 1975. — £6.75
- A. Baker, Transcendental number theory**, X+147 pages, Cambridge University Press, 1975. — £4.90.
- R. Balbes—Ph. Dwinger, Distributive lattices**, XIII+294 pages, Columbia, Missouri, University of Missouri Press, 1975. — \$25.00.
- D. W. Barnes—J. M. Mack, An algebraic introduction to mathematical logic** (Graduate Texts in Mathematics, Vol. 22), IX+121 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1975. — DM 26,50.
- A. Beck, Continuous flows in the plane** (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 201), XI+462 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1974.
- Ch. Berg—G. Forst, Potential theory on locally compact abelian groups** (Ergebnisse der Mathematik und ihrer Grenzgebiete, Bd. 87), VII+197 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1975. — DM 59,—.
- L. D. Berkovitz, Optimal control theory** (Applied Mathematical Sciences, Vol. 12), IX+304 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1974. — DM 23,30.
- C. Blatter, Analysis I—III** (Heidelberger Taschenbücher, Bd. 151—153), XV+204, XII+180, XII+184 Seiten. Berlin—Heidelberg—New York, Springer-Verlag, 1974. — DM 14,80+ +14,80+14,80.
- G. W. Bluman—J. D. Cole, Similarity methods for differential equations** (Applied Mathematical Sciences, Vol. 13), IX+332 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1974. — DM 23,30.
- K. Bögel—M. Tasche, Analysis in normierten Räumen** (Mathematische Lehrbücher und Monographien. II Abteilung: Mathematische Monographien, Bd. 25), XII+383 Seiten, Berlin, Akademie-Verlag, 1974. — 75,— M.
- Th. Bröckner, Differentiable germs and catastrophes** (London Mathematical Society Lecture Note Series, 17), Translated by L. Lander, VI+179 pages, Cambridge University Press, 1975. — £4.00.