



A SPATIOTEMPORAL STOCHASTIC FRAMEWORK OF GROUNDWATER FLUCTUATION ANALYSIS ON THE SOUTH - EASTERN PART OF THE GREAT HUNGARIAN PLAIN

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Abstract

The current study was performed on a Hungarian area where the groundwater has been highly affected in the past 40 years by climate change. The stochastic estimation framework of groundwater as a spatiotemporally varying dynamic phenomenon is proposed. The probabilistic estimation of the water depth is performed as a joint realization of spatially correlated hydrographs, where parametric temporal trend models are fitted to the measured time series thereafter regionalized in space. Two types of trend models are evaluated. Due to its simplicity the purely mathematical trend can be used to analyze long-term groundwater trends, the average water fluctuation range and to determine the most probable date of peak groundwater level. The one which takes advantage of the knowledge of expected groundwater changes, clearly over performed the purely mathematical model, and it is selected for the construction of a spatiotemporal trend. Model fitting error values are considered as a set of stochastic time series which expresses short-term anomalies of the groundwater, and they are modelled as joint space-time distribution. The resulting spatiotemporal residual field is added to the trend field, thus resulting 125 simulated realizations, which are evaluated probabilistically. The high number of joint spatiotemporal realizations provides alternative groundwater datasets as boundary conditions for a wide variety of environmental models, while the presented procedure behaves more robust over non-complete datasets.

Keywords: Monte Carlo simulation, stochastic estimation, spatiotemporal modelling, groundwater, climate change

INTRODUCTION

The water body closest to the surface and mostly above the first confined zone is called shallow groundwater (hereinafter groundwater in this study). Groundwater is a characteristic environmental element of alluvial lowlands. It is important on the one hand as a water source during the dry season for woody, as well as for deep-rooted vegetation, on the other hand, the vertically seeping groundwater means an important recharge for deep-water resources. Climate variability can have significant impacts on the groundwater (Treidel et al., 2012). These impacts are likely to be particularly severe when the watershed is located on an area that is predominantly disposed by the effects of altering environmental conditions, such as the Great Hungarian Plain. As a consequence of climate change the annual distribution of the precipitation tends to become more and more extreme (IPCC, 2013). These harmful processes can be enhanced by other stresses on the hydrological system, as may occur where there are large extractions from the watershed. Groundwater is vital to both environment and society, thus understanding how climate change and human activities may disturb regional water regime is exceptionally important for mitigating future water resources.

The approximate state of the groundwater can be estimated by observing a temporal sequence of the groundwater depth via an observation network operating over the

study area. However, the design of the observation network is critical and usually not enough dense to provide a clear view. Even the wells of the Hungarian measurement system, which can be considered much denser compared to other sensing networks worldwide, are located to cover 20-40 km² each. Thereby it is extremely important to select a proper method which enables accurate estimations of the groundwater state even from limited information of the measured sequences.

Previous studies (Pálfai, 1994) have revealed that in the recent decades the groundwater of the western part of the Great Hungarian Plain, the so called “Sand Ridge” between the Danube and the Tisza rivers is particularly affected by groundwater changes (predominantly the discharge of water resources). Although these former studies reached different results regarding the ultimate causes, our GIS-based quantitative analysis (Rakonczai and Fehér, 2015) successfully proved that the groundwater discharge is mainly influenced by altered climatic elements.

Our current study set both theoretical and practical goals. First, a brand new stochastic approach for the trend and variability analysis of the groundwater regime is introduced, considering water table as a spatiotemporally dynamic phenomenon. Second, we attempt to quantify the groundwater changes of the recent decade in South-Eastern Hungary.

STUDY AREA

The groundwater time series used in the current study consist of daily measured (depth to surface) water levels over the study site. The data were provided by the Lower Tisza District Water Directorate. The analysis is performed on an approximately 8490 km² study area in the South-Eastern part of the Great Hungarian Plain.

Groundwater depth measurements (given in cm) are available from 207 monitoring wells between 1/1/2005 and 17/5/2015. The spatial density of the stations shows locally diverse spatial distribution (Fig. 1). The length of the time series varies from 382 to 3,779 days. Most of the hydrographs has at least three daily measurements, only 5 of them show huge amount of missing data. There is no significant relationship between measurement density and spatial distribution. The area consists of three significantly dissimilar units.

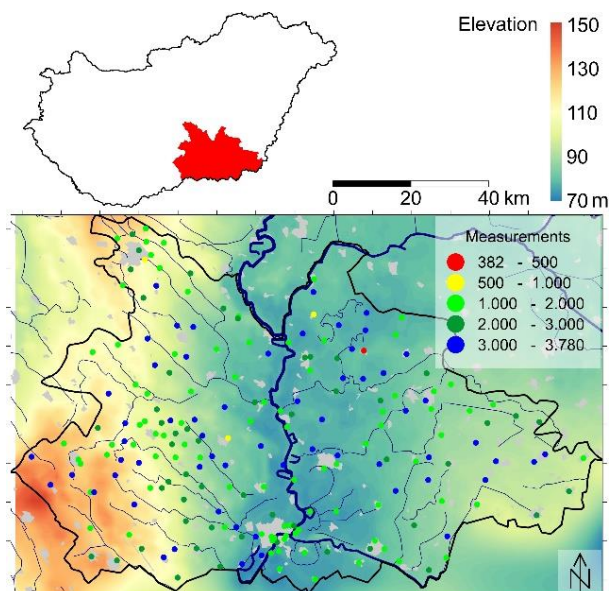


Fig.1 The study site with the operating gauge network alongside the count of available data

The western side is the eastern part of the so called Sand Ridge. This area is significantly affected by regional scale groundwater shortage in the last decades. Basin elevations range from 83 m on the Tisza valley to 143 m on the southwest. The area is deposited by the River Danube (the process was completed approx. 20–30 thousand years ago), and most of the landscape is covered by several-meter thick blown sand. Thereby, in the depressions semi-bound sand dunes, salty meadows can be usually observed on the surface. According to the dominant wind direction, the characteristic NW towards SE flow direction of the surface waters is determined by these relatively shallow features. These beds, however, barely carry any water, even in longer time periods.

Due to the elevated geomorphological position of the Sand Ridge, it is important to mention that groundwater resources can only be recharged by precipitation, since the opportunity of neither surface watercourses nor subsurface water flows is possible. This is the main cause of that more intense groundwater shortage which was observed in the higher area since the end of the 1970's when the amount of the annual rainfall significantly dropped for

one and a half decade. The typical discharge was around 2 metres, however on higher regions this could surpass 15 metres. In the higher areas this unfavorable situation could not be normalized even after extensive rainy periods. In the lower parts of the ridge however, the groundwater not just recovered from the water shortage, but the water level even increased thus sometimes caused harmful surface floodings after persistent rainy periods.

The eastern part of the study site belongs to the alluvial fan of the river Maros. Basin elevations vary between 80 – 108 m. The landscape is slightly sloping towards west (to the river Tisza) and the north (to the river Körös). The former (approx. 2 – 18 thousand years old, Sümeghy, 2015) river branches can be still observed more or less. Most of the area is constituted by good quality chernozem soils, but in the deeper parts alkaline vegetation is also common. Although groundwater is recharged primarily by precipitation, in this case the subsurface groundwater flow from higher regions plays significant role, through coarser, porous channel deposits of the former river beds encompass the area.

The River Tisza flows southwards in the axis of the some 10-12 km wide central region. This landscape is a young alluvial plain, which was continuously built by the Tisza with its sediments until in the middle of the 19th century the river regulations were completed. This area is the deepest part of the Great Hungarian Plain (as well as Hungary), ranging from 75.8 m to 83 m. In the last one and a half centuries the river flows between dams. Occasionally the groundwater is connected to the Tisza but in common years, (when both low water levels and floods also occur) this kind of effect can be shown only within 5 km to the river.

GROUNDWATER AS A DYNAMIC PHENOMENON

Estimations of large scale and long groundwater hydrographs are important boundary conditions of numerous environmental models. However, no sufficient, continuous information is available about the state of the groundwater table, thereby it has to be determined.

Various forms of geostatistical approaches have been developed to map groundwater table from scattered water-observations solely (Delhomme, 1978; Aboufassi and Marino, 1983; Neuman and Jacobsen, 1984; Dunlap and Spinazola, 1984; Fehér, 2012). The first attempt which involved topography as supplementary data is based on the full cokriging system (Hoeksema et al., 1989), and was later improved to solve multiple usability problems (Xu et al., 1992; Xianlin and Journel, 1999). Kriging with an external drift (KED) provides an alternative to apply soft data into groundwater estimation (Desbarats et al., 2001; Fehér, 2015). However, kriging with local means (Goovaerts, 1997), which is based on undermining the global regression of the groundwater and soft data, offers the least estimation error on some Hungarian study sites (Fehér, 2015; Geiger, 2007).

The above mentioned studies focus solely on the spatial autocorrelation of an available, contemporaneous data thus consider interaction neither on temporal (within

groundwater time-series on every single wells) nor on spatiotemporal (between spatially correlated hydrographs) domain. Commonly, the rarer the monitoring network is or the longer the missing data period is, the more important the selection of the appropriate estimation method is.

Global structures of autocorrelation functions, thereby results of any geostatistical estimation are determined by (1) the spatial and/or temporal sampling structure and (2) the stationary covariance function that characterize the relationship between any available observation (Journel, 1989). Resource estimations and analyses of dynamic natural processes are usually investigated by comparison of spatial patterns at different time instants (eg. Malvic and Zelenika 2014). However long-term observation networks rarely if ever provide complete datasets, thus (1) covariance matrices of different sampling patterns differ significantly even if similar covariance functions are assumed and (2) ignorance of the local behaviour of the temporal process or a temporary extreme event (such as locally altering intensity of human activities) will cause significantly different estimation from the expected local process, thus result in false resource estimation (Fehér, 2015). Though the observations are neither temporally nor spatially unique, the interpolation with missing data causes modification on the covariance matrix of any selected kriging system. This leads to a somewhat semantic error, while attempting to relate groundwater levels at two different time instants. Consequently, groundwater maps, which are estimated with distinct data distribution are incomparable, hence inadequate for resource or flow analysis. Furthermore, the application of these data as boundary condition of any type of numerical models, leads to false outcomes.

Numerical models are doubtlessly able to increase reliability of water table estimations by exactly defined physical relationships between groundwater and its influencing factors. However, these approaches are computationally expensive, furthermore uncertainties of model parameterization, temporally incomplete knowledge of the varying environmental factors, and dynamic upscaling of different input datasets make difficult the application of such numerical models (Fehér, 2015). In addition, the influencing factors of groundwater level proceed both temporally and spatially at different scales, so any spatial or temporal averaging or dynamic upscaling of these phenomena would change the original spatiotemporal correlations (Kyriakidis and Journel, 1999).

Groundwater, however, can be characterized in a stochastic framework (Mucsi et al., 2013), by involving soft data into the estimation (Fehér, 2012, 2015; Geiger, 2015), like digital elevation models, even if these models are not capable to explicitly represent dynamically changing relationships between groundwater estimate and a particular influencing process.

Groundwater time series commonly exhibit both spatial and temporal autocorrelation. Early spatiotemporal estimations treated time simply as an additional (2D+1) dimension (Eynon and Switzer, 1983). But, even if spatial autocorrelation is considered constant at any time instant, temporal autocorrelation may significantly differ place to

place, due to different local influencing factors. Consequently, conventional interpolation approaches (including any type of 2D kriging algorithm) are inappropriate to handle this kind of joint spatiotemporal autocorrelation.

Since the temporal domain of the groundwater observations is more densely sampled compared to the spatial domain, the estimation of missing data can be more reliably carried out on the hydrographs. The well-informed time domain can be easily exploited by either employing multivariate AR models (Hosung, 1994; Rétháti, 1977a, b), generalized linear models (Gotway and Stroup, 1997) or spectral analysis (Chatfield, 1996; Kovács et al., 2004, 2011), though the results cannot be interpolated over the entire domain (Rouhani et al., 1992), thus these kind of spatial analysis are inappropriate to model groundwater change.

The spatiotemporal variability of the groundwater cannot be accurately characterized via the above mentioned, purely deterministic models, due to the deficient knowledge of the both spatially and temporally altering physical environment and limitations of observation capabilities. Since model parameters (like land use, hydrometeorological conditions, irrigation, pumping, instant state of the canal network etc.) are varying over both space and time (see Fehér, 2015), long term hindcast type deterministic models, which consider these influencing factors static, would make the false sense in the modeller that his all-rounder model is perfect.

In this paper groundwater is considered as a joint realization of a set of spatially correlated time series Kyriakidis–Journel (2001a, b), one for each grid node. Hydrograph model parameters are varying in space thus capturing space-time interactions. The difference of observed groundwater depth from these local trend estimates are regarded as a realization of a stationary spatiotemporal stochastic process.

Geostatistical space-time models provide a framework based on probability theory to analyze and forecast through spatial and temporal autocorrelation among available observations. In most cases a stochastic model by its own cannot explain unusual anomalies of the datasets, although that can often be explained by some other influencing physical processes.

The modelling of such dynamic processes brought various, but somehow similar approaches for numerous scientific and engineering fields including hydrometeorological conditions (Armstrong et al., 1993), precipitation profiles (Kyriakidis et al., 2004), pressure-temperature relationships (Mardia and Goodall, 1993; Hottovi and Stechmann, 2015), variability of geophysical components (Handcock and Wallis, 1994; Bogaert and Christakos, 1997a). Other similar approaches were implied in the estimation of soil moisture (Goovaerts and Sonnet, 1993; Papritz and Flührer, 1994; Heuvelink et al., 1997), diseases (Christakos–Hristopoulos 1998), environmental hazards (Mateua and Ignaccolo 2015), deposition of atmospheric particles (Kyriakidis, 2001b; Singh and Gokhale, 2015) and ecological dynamics (Hohn et al., 1993; De Iaco et al., 2015).

These procedures are common in the sense that they separate the dynamic phenomenon into (1) spatiotemporal “trend” component, which expresses some long term patterns and (2) the “residual” component that represents

some higher frequency fluctuations around the trend component. The separation of the trend and residual component reflects a subjective decision of the modeler and the available knowledge of the phenomena. The temporal trend can be characterized by deterministic parameters for each gauge (Dimitrakopoulos and Luo, 1997). The spatial correlation between temporal parameters represents every spatiotemporal interaction that exists between all of the influencing factors. The residual component is induced by some small scale, rapid or interim intense effects.

Several studies quantified long term hydrograph patterns of the Great Hungarian Plain. Recently performed time series analyses indicated annual periodicity on about 97% of the gauges (Kovács et al., 2004, 2011), which is clearly explained by annual discharge-recharge cycle (Ubell, 1954). Another questionable 5 and 11 year-long periodicities were also designated, which might slightly be identified by multiannual periodicity of the precipitation (Kovács and Turai 2004). Another 14-17 year-long or even longer periodicities are also detected on a smaller set of hydrographs (Rónai, 1985; Rétháti, 1977), but the volume of these effects are negligible. It is important to highlight that despite these longer periodicities are “mathematically” really spotted, yet no clear, physical explanation of the underlying causes can be revealed, consequently, multi-annual periodicities must be treated carefully.

The temporal profiles are spatially non-stationary, due to altering spatial intensity of the environmental factors. Closer to rivers hydrograph trends usually show fewer long-term changes (Bezdán, 2011) and prone to mimic water level of the nearby river. The magnitude of the annual groundwater fluctuation on steeper terrain is less than close to permanent watercourses or even on elevated reliefs. The primary reason for the latter is that groundwater continuously replenished by gravitational flow through porous media from the ridges. By contrast, the only water source is the periodically intense rainfall in elevated areas (Rakonczai, 2014).

Meteorological conditions show significant annual and local variability in the Hungarian Plain, so groundwater trends can be hardly expressed by polynomial and sinusoidal functions. Long-lasting river level changes, alteration of land use, irrigation and several other factors trigger further high frequency, hardly modelable effects on the groundwater level (Négyesi, 2006). Therefore, it is very crucial to take the patterns of the non-stationary spatial variability of the time series into account.

THE SPATIOTEMPORAL FRAMEWORK

Intrinsic hypothesis of the Simple Kriging requires a stationary variogram model, so that elimination of any relationship between groundwater observations and the spatiotemporal coordinates (Deutsch and Journel, 1998). Comparison of resources of non-complete groundwater datasets may undermine the strong correlation ($\rho = 0.98$) between groundwater elevation (above sea level) and topography (Fehér and Rakonczai, 2012; Geiger, 2015). However, the regression function can also be interpreted as a locally varying spatial trend (Fehér, 2015), thus only the residuals of the regression model need to be estimated

(Goovaerts, 2000). Instead of the spatial regression residuals, hereinafter, for practical considerations, the groundwater depth (to surface) is considered spatially detrended.

Groundwater time-series are decomposed into spatial and temporal trend components (Eq. 1) and the fitting error of the trend function. Thereafter the trend components express TOPO spatial as well as $E(Z_u t) = m_u$ temporal, smoothly varying expected patterns, and the rest is considered a frequently changing, stationary $E(R_u t) = 0$ residual component.

$$Z_u(t) = \text{TOPO} + m_u(t) + R_u(t) \quad (\text{Eq. 1})$$

Hydrograph trends can be expressed via $(K + 1)$ deterministic functions, which are defined by an initially chosen subjective model decision (Dimitrakopoulos and Luo, 1997). Most of the inspected hydrographs shows a downward trend, which was modelled via versatile polynomial functions. Spatial patterns of each estimated b_k trend coefficient theoretically expresses the locally and temporally varying intensity of some (known or unknown) f_k background process.

The spatial distribution of these partial trend component intensities can be expressed in two steps. First, a general function (Eq. 2) needs to be fitted individually on each hydrograph, then the given function parameters can be spatially estimated, considering their variograms.

$$m(\mathbf{u}_\alpha, t_i) = \sum_{k=0}^4 b_k(\mathbf{u}_\alpha) f_k(t_i) \quad t_i \in T_\alpha \quad (\text{Eq. 2})$$

The spatial consistency for correct reconstruction of the trend field require either (1) to use identic variograms for each component regionalization or (2) to apply a Linear Model of Coregionalization (Deutsch and Journel, 1998; Fehér, 2012) or (3) to perform the estimation on an orthogonalized system, applying dimension reduction. In this study the Principal Component Analysis was applied for the orthogonalization of the trend components. A set of 125 sequential Gaussian simulations (Deutsch and Journel, 1998) of temporal trend PCA-values were generated on a 156×106 grid with a cell size of $1 \times 1 \text{ km}^2$ for ensemble prediction. The number of realizations was determined by the spatial analysis of thickness change of the confidence intervals. The trend field for the entire spatiotemporal domain was thereafter revealed for each (\mathbf{u}_α, t_i) grid node using (Eq. 2), since estimated b_k trend coefficients are given as a median type estimation of the inverse PCA transformation of each simulated random field PCA score realization.

Since simple kriging is an exact interpolator, each trend estimate preserves its initial value at the exact gauge locations. Consequently, regionalized trend ensemble values at each gauge are quasi identical to the value provided by the fitted trend function at any time instant. In addition, the station specific, detrended, zero mean $R_u(t)$ residual process is quasi identical to the exact fitting error of the trend model. The ultimate goal of the spatiotemporal residual simulation is to estimate these values.

The first step is, to estimate spatial distribution of the temporal autocorrelation parameters (nugget, sill and range) individually for each variogram parameter, thus each residual time series is divided by its standard deviation thereby proofing its standard normal distribution that enables Simple Kriging to be used in a Sequential Gaussian Simulation

framework (Deutsch and Journel, 1998; Mucsi et al., 2013). The standard deviation values of the calculated residual sequences were then simulated over the study site. The obtained residuals were back-transformed into the original data dimensions only after the stochastic simulation.

For the sequential Gaussian simulation, a spherical variogram function was automatically fitted individually on each normally distributed residual time series. The obtained nugget, sill and range parameters were then co-regionalized over the study site, by considering their spatial relationships. Here we chose a simple full cokriging method for the estimation (Goovaerts, 2000). In order to determine spatial autocorrelation of the gauges, the spatial variogram of the residuals was also modelled, via cross-validation (Isaaks and Srivastava, 1989). Thereafter the residuals were considered as joint-realization of a spatiotemporal stationary process. Subsequently sequential Gaussian simulation (Deutsch and Journel, 1998) was performed at each spatial coordinates with the appropriate spatial and temporal variogram parameters, thereby 125 temporal profiles were obtained at each unsampled grid node.

As the realizations of the residuals are obtained via conditional Sequential Gaussian Simulation and added to the estimated trend ensembles, 125 geostatistical spatiotemporal distributions of daily groundwater levels were revealed. The $S = 125$ alternative, equiprobable realization enables the probabilistic characterization of both the regionalized groundwater and also some simulated trend components. The following indicators were applied in the current study:

(a) At each node \mathbf{u} the $\bar{z}(\mathbf{u})$ E-type estimate (the arithmetic mean) of the S simulated values $z^{(s)}, s = 1, \dots, S$ is given as:

$$\bar{z}(\mathbf{u}) = \frac{1}{S} \sum_{s=1}^S z^{(s)}(\mathbf{u}), \quad \mathbf{u} \in D \quad (\text{Eq. 3})$$

(b) At each node \mathbf{u} the $\sigma(\mathbf{u})$ local standard deviation of the S simulated values $z^{(s)}, s = 1, \dots, S$ is written as:

$$\sigma(\mathbf{u}) = \sqrt{\frac{1}{S} \sum_{s=1}^S [z^{(s)}(\mathbf{u})]^2 - \bar{z}^2(\mathbf{u})}, \quad \mathbf{u} \in D \quad (\text{Eq. 4.})$$

(c) As E-type values and standard deviations of any individual grid node distributions are already revealed, the $Prob_c(\mathbf{u})$ confidence interval can be calculated in any c significance level. Thicker confidence interval indicates higher estimation uncertainty:

$$Prob_c(\mathbf{u}) = \bar{z}(\mathbf{u}) \pm Z_{c/2} \times \frac{\sigma(\mathbf{u})}{\sqrt{S}} \quad (\text{Eq. 5})$$

(d) $c = 0$ significance level of the confidence interval reveals the median-type estimate of the grid node estimation

$$Md(\mathbf{u}) = \bar{z}(\mathbf{u}) \pm Z_0 \times \frac{\sigma(\mathbf{u})}{\sqrt{S}} \quad (\text{Eq. 6.})$$

(e) The local probability $p_k(\mathbf{u}; z)$ of the grid node estimation exceeds a given threshold can be revealed as:

$$p_{(\mathbf{u}, z)} = \frac{1}{S} \sum_{s=1}^S i_k^{(s)}(\mathbf{u}, z), \quad \mathbf{u} \in D \quad (\text{Eq. 7})$$

where $i_k^{(s)}(\mathbf{u}; z)$ is the indicator of $z^{(s)}(\mathbf{u})$, its value is: $i_k^{(s)} = 1$, if $z^{(s)}(\mathbf{u}) > z$, otherwise 0.

RESULTS

The groundwater level is simulated as joint ensemble images of a set of spatially correlated time series. Each hydrograph is decomposed into a spatiotemporal trend and a spatiotemporal residual component. The analysis proceeds by first comparison of goodness of fit of different parametric trend functions, followed by the regionalization and spatial evaluation of the revealed trend coefficients and long term groundwater alteration on the site.

Dimitrakopoulos and Luo, (1997) proposed three types of trend models which can be applied for time series modelling. The considered models satisfy both the tensorial invariance condition and provide a unique solution for the kriging system (Deutsch and Journel, 1998). These models are constituted by the sum of different degrees of polynomial and Fourier components. In this study we compared each combination of models from first to third degree polynomial and first to third degree Fourier models. The resulting mean average fitting error is ranging from a significant 41 to 50 cm. Our analysis highlighted that the increase of the components does not necessary result a significant increase of the fitting performance.

In this study we developed another model which involve some temporal pattern in the fitting process. The latter provided a 16-18 cm matching error, depending on the degree of the applied model parameters. Consequently, the latter is more suitable to express local dynamics of groundwater changes. Despite the significant matching error, the former model is still appropriate to interpret long-term differences of the groundwater dynamics.

Trend modelling without a temporal pattern model

Spectral analysis (Kovács et al., 2004, 2011) indicated annual cycle as dominant periodicity on the individual time series. Among analyzed function structures the first order polynomial (linear) function (Eq. 8) with annual periodicity provided besides very good function match, an easily interpretable result.

$$m(\mathbf{u}_\alpha, t_i) = b_0(\mathbf{u}_\alpha) + b_1(\mathbf{u}_\alpha)t_i + b_2(\mathbf{u}_\alpha) \cdot \cos\left(\frac{2\pi}{365.25}t_i\right) + b_3(\mathbf{u}_\alpha) \cdot \sin\left(\frac{2\pi}{365.25}t_i\right)$$

$$m(\mathbf{u}_\alpha, t_i), i \in T_\alpha \quad (\text{Eq. 8})$$

where $b_0(\mathbf{u}_\alpha)$ characterizes the initial depth of the groundwater on 01/01/2005, whereas $b_1(\mathbf{u}_\alpha)$ expresses long term recharge or discharge of the groundwater in the surroundings of any individual gauge.

The $a(\mathbf{u}_\alpha)$ amplitude of annual periodicity express an expected groundwater fluctuation range and the $\phi(\mathbf{u}_\alpha)$ phase value designates the serial number of the date concerning to the highest groundwater level. These indicators are averaged over the considered 10-year period and both are expressed by $b_2(\mathbf{u}_\alpha)$ and $b_3(\mathbf{u}_\alpha)$ Fourier components:

$$a(\mathbf{u}_\alpha) = \sqrt{[b_2(\mathbf{u}_\alpha)]^2 + [b_3(\mathbf{u}_\alpha)]^2} \quad (\text{Eq. 9})$$

$$\phi(\mathbf{u}_\alpha) = \tan^{-1} \left(\frac{b_3(\mathbf{u}_\alpha)}{b_2(\mathbf{u}_\alpha)} \right) \quad (\text{Eq. 10})$$

The $b_k(\mathbf{u}_\alpha)$ intensity coefficients for each gauge were calculated individually by Ordinary Least Squares algorithm. Note that the selected ordinary least squares (Searle, 1971) algorithm is a subjective model decision which completely determines the trend component estimation. The resulting components are thereafter regarded as exact data and estimated individually by sequential Gaussian Simulation (Deutsch and Journel, 1998; Mucsi et al., 2013) and evaluated later. The analysis of the correlation does not indicate any significant relationship among component pairs, unless some quasi relation between long term linear trend and periodicity. In practice this may indicate the relationship between low annual periodicity and a deep water level or high annual periodicity with lower altitude.

Trend analysis and stochastic simulation of the groundwater using a temporal pattern

The former chapter pointed out that elementary polynomials and sinusoidals provide poor efficiency in representation of frequent and uncommon hydrograph divergences by annually predicted periodicity and long term trends. In the forthcoming chapter a more accurate approach is presented. This chapter is based on the theory that a set of influence factors that affect large scale trend of the groundwater, are represented on every single hydrograph, while their intensity is varying over the site under study. The analysis begins with modelling of a common profile. This profile describes the most likely pattern, thus resulting minimal fitting error on most of the time series.

First, this profile can be a particular auxiliary data, for example satellite gravimetry (Gravimetry Recovery and Climate Experiment – GRACE, eg. Wardlow et al., 2016). Second, an intrinsic solution might be a series of arithmetic means or medians of time instants as it is presented hereafter. The joint time series model can be calculated in three subsequent steps:

1. Shifting every single time series by its mean value (normalization)

2. Dividing every single time series by its standard deviation (standardization)
3. Eventually, standard-normal values of different time series that belong to the similar time instant are considered as a statistical set, whose arithmetic mean or median can be calculated. The series of these values constitute the finally established joint time series model (Fig. 2).

The obtained joint model thereafter can be successfully fitted to the majority of hydrographs, using affine transformations (rotation, translation, scaling) and Fourier modulations (phase shifting). Note that arithmetic mean is more sensitive to outliers that are likely present in the dataset, additionally median is a more robust method if the statistical distribution of a set is non-normal. This way median over-performs the arithmetic mean, even if it does not appear on the fitting error comparison significantly.

The task is then to determine proper transformation parameters for every single \mathbf{u}_α gauge:

$$\begin{aligned} m(\mathbf{u}_\alpha, t_i) &= \sum_{k=0}^K b_k(\mathbf{u}_\alpha) f_k(t_i) \quad t_i \in T_\alpha \\ &= b_0(\mathbf{u}_\alpha) + b_1(\mathbf{u}_\alpha) Md(t_i) + b_2(\mathbf{u}_\alpha) t_i \\ &\quad + b_3(\mathbf{u}_\alpha) \cdot \cos\left(\frac{2\pi}{12} t_i\right) + b_4(\mathbf{u}_\alpha) \\ &\quad \cdot \sin\left(\frac{2\pi}{12} t_i\right) \end{aligned} \quad (\text{Eq. 11})$$

where $b_0(\mathbf{u}_\alpha)$ indicate translation by y-axis, $Md(t_i)$ equals to the i^{th} value of the above discussed joint median time series model (Fig. 2). The joint time series is modulated by b_1 intensity along y-axis. $b_2(\mathbf{u}_\alpha)$ parameter defines the rotation of the temporal trend profile, $b_3(\mathbf{u}_\alpha)$ and $b_4(\mathbf{u}_\alpha)$ expresses the phase shifting along x-axis, thereby adjusting station specific differences of the annual groundwater phase. Note that higher order polynomials do not increase fit performance significantly, but their application would unnecessarily complicate the analysis.

The $b_k(\mathbf{u}_\alpha)$ intensity coefficients were calculated for each gauge individually by non-linear least squares (Gauss-Newton) algorithm. Note that the selected non-linear least squares (Searle, 1971) algorithm is a subjective model decision thereby completely determines the trend component estimation. The resulting components are thereafter regarded exact data. Although



Fig. 2 The applied joint median type time series model compared to a satellite gravimetry sequence (dimensionless units)

a very good fitting performance is achieved, the temporal trend model does not represent each hydrograph similarly. The proportion of the explained standard deviations (Fig. 3) shows that some hydrographs can not be modelled well locally. In these cases, the residual model will express the proportion that does not fit to the local trend.

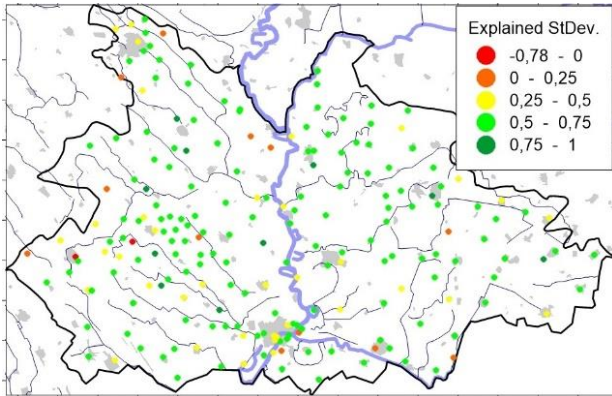


Fig. 3 Proportion of the explained standard deviation by gauge
Mathematical interpretation of the trend functions

The study proceeded by analysis of three characteristic hydrographs (Fig 4.). Limited information is available on Kecskemét gauge at the initial period. This well is located in an elevated area of the Sand Ridge, close to an urban area. In the initial period the groundwater seems shallow related to other time instants, until it unexpectedly drops

by 4 metres in 2007. This deep state does not change until an extreme precipitation period in 2010. It can be also recognized, that the water level does mimic long range trend pattern only when it is not deeper than 150 cm. Csánytelek metering station follows the global trend until the end of 2010, thereafter the groundwater suddenly drops by 80 cm, while the hydrograph shape is still almost identical to the global trend model. Probably the precipitation this time still recharges groundwater with unchanged intensity until the water level is restored to the global trend in the winter of 2012—2013. The sudden discharge might be due to some anthropogenic inventions around the gauge. The well near Hódmezővásárhely shows uninterrupted groundwater trend. Analyzing long term trends, Hódmezővásárhely station is permanent, Kecskemét observator indicates increasing whereas the Csánytelek station shows a downward trend. Weak periodicity can be recognized in Kecskemét well in contrast to the other gauges introduced.

As shown in Fig 4. both fitting approach can represent long term tendencies just fine, whereas median based matching can reproduce hectic patterns of the groundwater dynamics better. While annual periodicity can somehow be reproduced, sudden uplifting of the groundwater around 2010 and 2011 can be hardly handled by pure Polinomial – Fourier models, due to extreme precipitation conditions (Szalai et al., 2012). The approach implementing a median type temporal pattern provides definitely a more sophisticated solution

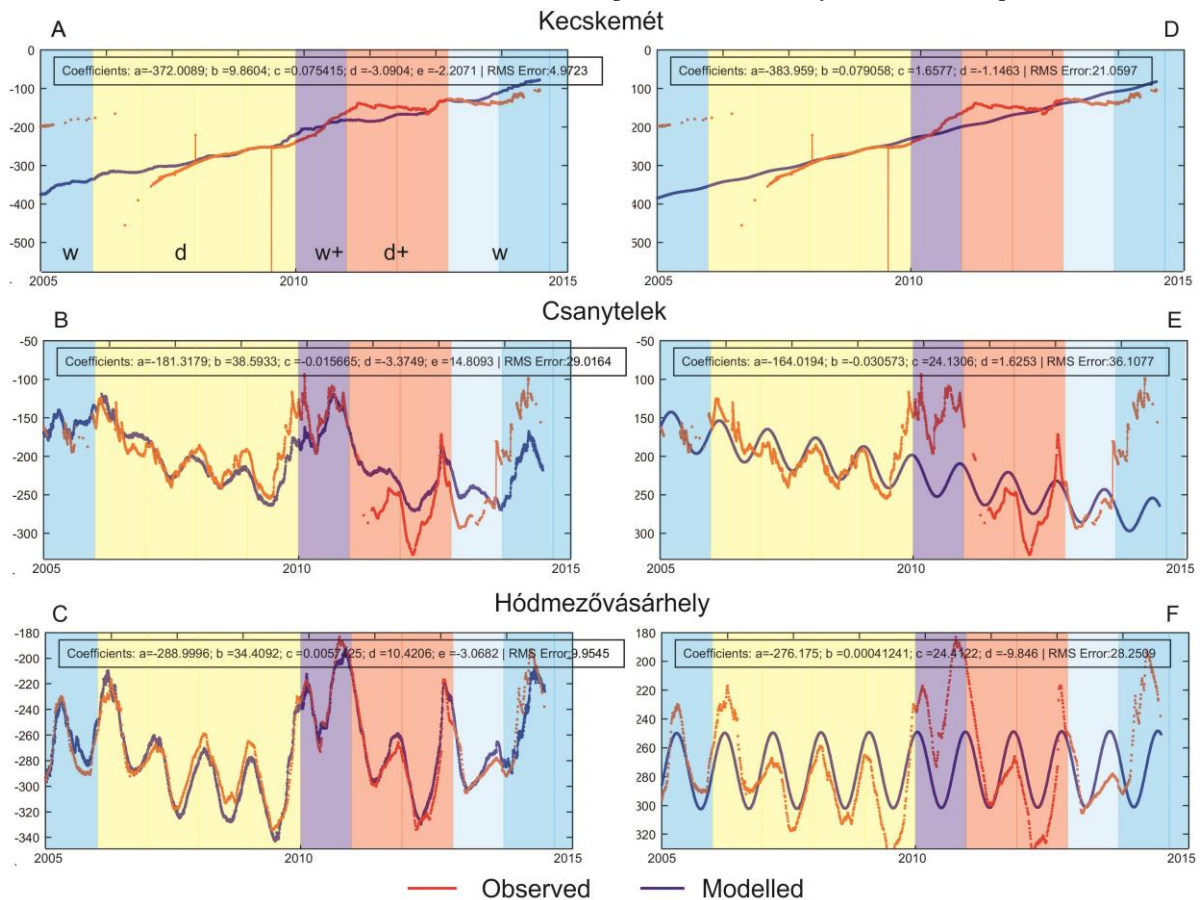


Fig. 4 Comparison of some characteristic groundwater time series (red) and their calculated temporal trend functions using purely mathematical (right column) and median type temporal pattern (on the left). W and D indicate wet or dry periods, whereas + means an extreme event.

compared to the one based on solely mathematical functions. Consequently, the median type model is very efficient in the estimation of missing observations.

By contrast, the purely Polynomial – Fourier type method is capable to interpret long-term (10 year) average groundwater tendencies for each individual gauge. The determined trend model parameters differ from gauge to gauge which enables to estimate long-term groundwater tendencies, by spatial interpolation of the appropriate model parameters. Through the analysis of the 1-year-long trend of every single observation of a hydrograph is represented by similar weight, consequently the fitted trend is not too sensitive to a small amount of outliers, rather more to the long period of missing data. In the present case the main analyzed factors (slope, fluctuation range and the date of the typical date of the highest water level can be determined by these restrictions.

Spatiotemporal estimation of the groundwater level

Spatiotemporal estimation of the groundwater is performed by sequential Gaussian simulation (Deutsch and Journel, 1999; Mucsi et al., 2013) of median model parameters. 125 equiprobable realizations were generated thus enabling geostatistical evaluation of the results. Cross covariance matrix of the given b_k parameters (not presented) highlighted some discrepancy among parameters, thus principal component analysis was performed to project them into x_k orthogonal planes. In the forthcoming step a spatial estimation of the components was reconstructed from these orthogonal planes, and the values of the appropriate grid node alongside temporal coordinates were substituted into Eq. 11. The analysis is proceeded by spatiotemporal geostatistical simulation of residual components. The temporal hyperparameters are modelled automatically via spherical model and regionalized by simple cokriging. The spatial variogram parameters are modelled via cross-validation.

The stochastic component of the groundwater at most cases characterized by at least a half-year-long temporal range, which represents the length of the autocorrelation within the hydrograph. Consequently, a half-year-long sampling interval would be enough to make good estimations on the stochastic origin changes of the groundwater.

The relatively small standard deviation values indicate that the applied temporal trend function was very

effective, whereas the low value of temporal sills represents the portion of the autocorrelation function which can be expressed by the applied spherical model. The higher volume means the less randomness of the time series.

Eventually, 125 equiprobable realizations of time series were generated for each spatial grid node using sequential Gaussian simulation (Kyriakidis, 2001a). The results are added node-by-node to the Median-based temporal model realizations. These joint realizations were finally evaluated geostatistically by Eq 3. – Eq. 7. The multiple generated realizations made enable to probabilistically characterize (1) temporal change of volume, (2) estimation uncertainty, (3) spatial pattern, and (4) gravitational flow of the simulated groundwater and evaluate its environmental effects.

DISCUSSION

The investigated 2005-2015 period is very interesting from the hydrometeorological point of view. Since the beginning of the detailed meteorological measurements (more than 100 years ago), the two highest and also the lowest precipitation volumes have been measured during the analyzed period in Hungary. The spatial average of the arrived precipitation over Hungary was 938 mm in 2010, but just 407 mm in the subsequent year. At the beginning of the investigated period, the precipitation exceeded the multi-annual average (note that 2005 was the second rainiest year), but due to the preceding dry period, groundwater resources yet showed significant deficit (compared to preceding decades) on the investigated site (Fig. 5). During the four subsequent dry years (alongside the well-defined annual periodicity), the depletion of groundwater resources occurred. The volume of the water shortage on the study site is at least 3 km³.

In the very rainy year of 2010 the water shortage of the previous years not just recharged, but a significant water surplus was generated. Consequently, inland excess water occurred in multiple areas, as a result of the increase of the groundwater levels. Right after an extremely wet year as an effect of an extremely dry year, a significant groundwater shortage was generated again. 2013 and 2014 were again rainier than the average, thus groundwater resources recharged again to the stage where they were 10 years ago.

We found that groundwater discharge primarily affected the higher altitude areas in the preceding 40 years.

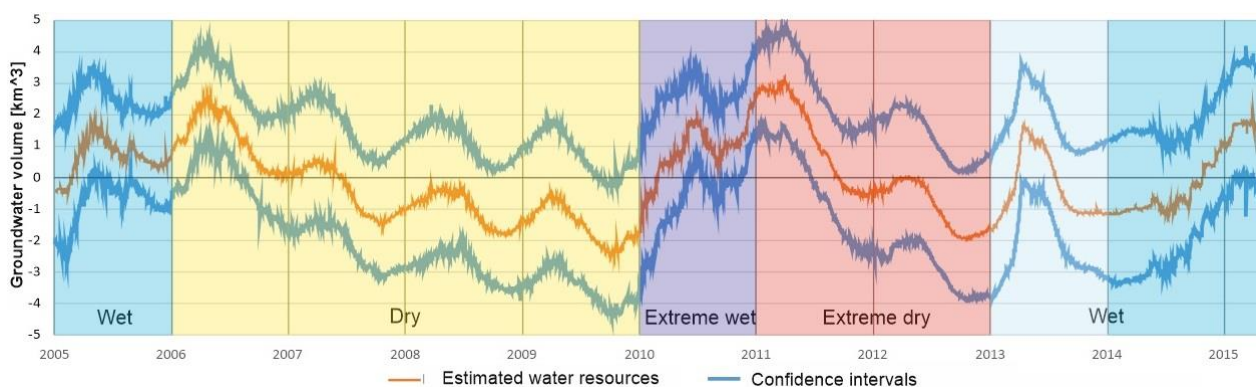


Fig. 5 Estimated groundwater resources related to the average of 2005-2015 versus precipitation related to the average of 1970-2000

However, we recognized that during the investigated 10-year-long time period, a downward tendency continued on the study site, but, at the same time, together with long-term trends, an exciting thing was observed: in the following years the depletion is more intensely affected the lower altitudes (Fig 6.).

At first sight this fact can be hardly explained, but it has a very clearly identifiable cause: in the recent years the characteristic spring and early summer floods of the Tisza River are absent. The river stepped out from its bed only once for a short time (in May 2013) between the spring of 2011 and the end of 2015. Nearly three years ago its water level was really deep, in spite of the Serbian river damming. The underground water flow probably increased towards the river, due to the permanent low water level. Due to the reverse flow direction in general case in a particular part of the year (when the Tisza is flooding), the groundwater resources can recharge some km wide along the river. This has importance not only because of the recharge of the water resources but also because of the tendency of minimal groundwater levels too. Indeed, if the groundwater level close to the river decreases, the underground water flow from several-meter high area towards the river possibly increases. This may explain the observed groundwater discharge on transient altitude zones.

A possible reason may arise the decreasing infiltration due to continuously developing wastewater sanitation system, however its effect should be more significant in a longer time period. Not surprisingly, due to the foregoing, extreme steep, a decisively downward trend can be experienced on the hydrographs along the River Tisza.

Locally such steep down- or upward trends can be experienced often, that hydrographs of the neighboring gauges absolutely do not confirm. These are probably consequences of a specific anthropogenic effect (like, exploitation of irrigation water from groundwater due to serious drought), long-lasting floods or inland excess water.

In the next step we analyzed average water level changes of the hydrographs from 2005 to 2015. Gauges located close to the Tisza River show the highest fluctuation within a year (Fig. 7.). On the alluvial fan of the Maros River slightly smaller, on the central area of the Sand Ridge some minimal water fluctuation can be determined. Note, that these values are averaged to the whole analyzed 10 years (Fig. 4 D, E, F), however interannual values significantly diverge from the multiannual average. The average water fluctuation range in the observed area mostly can be determined within a 10 cm confidence interval, however the highest and lowest uncertainties were also observed along the Tisza river. Note that the lowest uncertainty area around Szeged (a city lying in the southern central area) are probably affected by the effects of the urban area.

The average date of the highest groundwater level is also determined in the 10-year long time series. As awaited, this pike value can be expected in most cases in March and April (Fig. 8.), however it can be determined with high uncertainty (with 30-60 day wide confidence interval). At multiple locations significantly different pike time moments can be experienced. This may refer to water inundation.

Our analysis indicated that the least groundwater was stored on 12/10/2009, and the most was available on 24/03/2011 (Fig. 5). Accordingly, the water resource

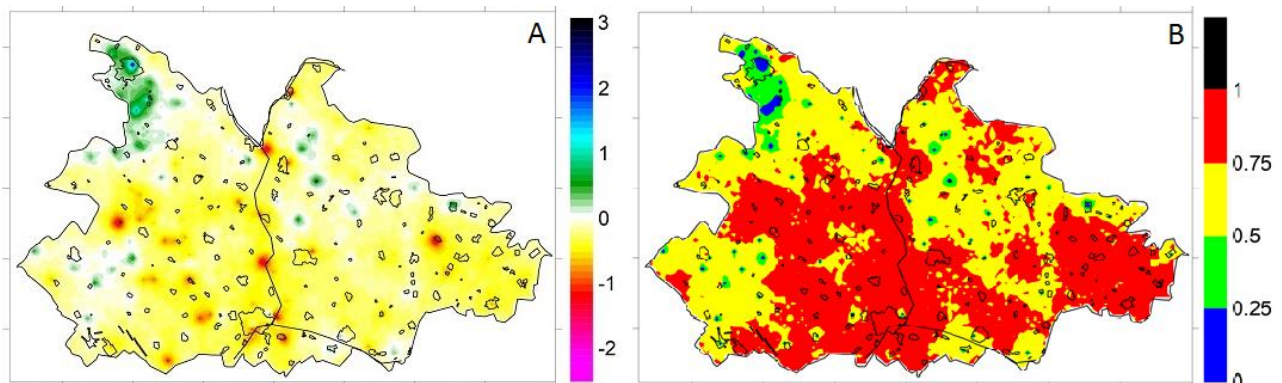


Fig. 6 Average annual change of groundwater in cm (A) and probability of the groundwater discharge over the 2005-2015 period (B)

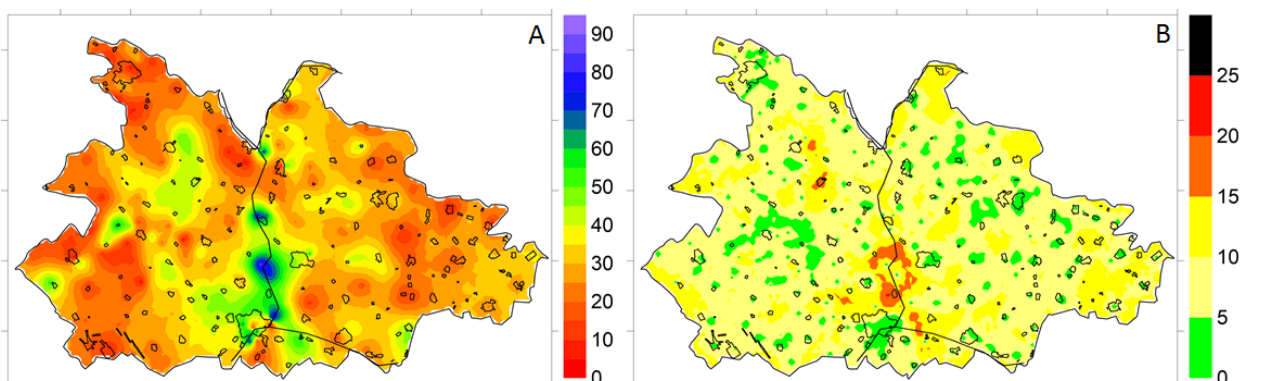


Fig. 7 Estimated groundwater fluctuation range averaged over 2005-2015 (A) and the estimation uncertainty expressed in cm (B)

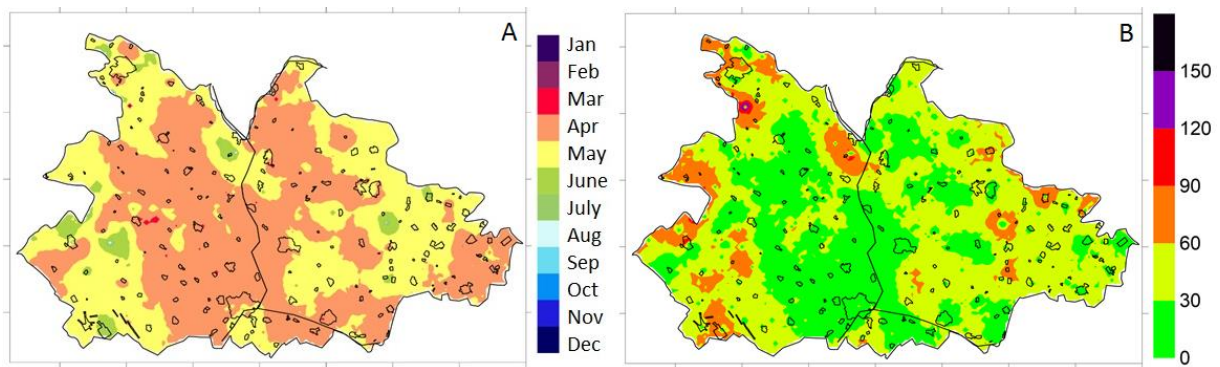


Fig. 8 Estimated date of the pike groundwater level averaged over 2005-2015 (A) and the estimation uncertainty expressed in days (B)

shows cca. 5.5 km^3 fluctuation (discharge) in one and a half years. The groundwater level map of these two moments is illustrated in Fig. 9. Extremely low groundwater level can be seen in 2009 on the western site, whose spatial pattern draws well the Sand Ridge. In contrast, the highest water level can be seen not along the Tisza River, as it was expected, but half way between the Sand Ridge and the river. This has two possible reasons: (1) the river plays a role as a “drainage channel”, thereby tapping the water resources from its neighborhood, and (2) the water resources are significantly replenished by gravitational flow from the Sand Ridge even in drought periods. The relatively higher water levels of the eastern part of the study site can be considered similarly as a result of uplifting underground flows through former water beds.

The situation in 2011 is at least as interesting as the above explained. On the area of the Sand Ridge some 2-3-meter-deep water levels evolved, which means 2-meter recharge there. However, from agricultural perspective a very harmful situation occurred. On most of the site, the water depth was shallower than 1 meter, and in worst cases

it inundated the surface. For example, in the eastern zone of the Sand Ridge, the subsurface water flow from the higher areas elevated the groundwater level insomuch that it caused floodings on the ground. The pattern of the groundwater on the former alluvial fan of the Maros river quasi designate the former main river beds, probably attributable to the the permeable bed deposits.

CONCLUSION

The methodology originally proposed by Kyriakidis and Journel (2001) has been improved to the stochastic analysis of non-complete, daily observed groundwater levels on a South-Eastern Hungarian site.

Groundwater hydrographs are considered spatially correlated due to the similar long-range background processes. The well-informed time domain was capitalized to improve reliability of a spatiotemporal estimation. The simulated hydrographs provide a probabilistic framework to estimate and evaluate both temporal and spatial patterns, furthermore, changes of

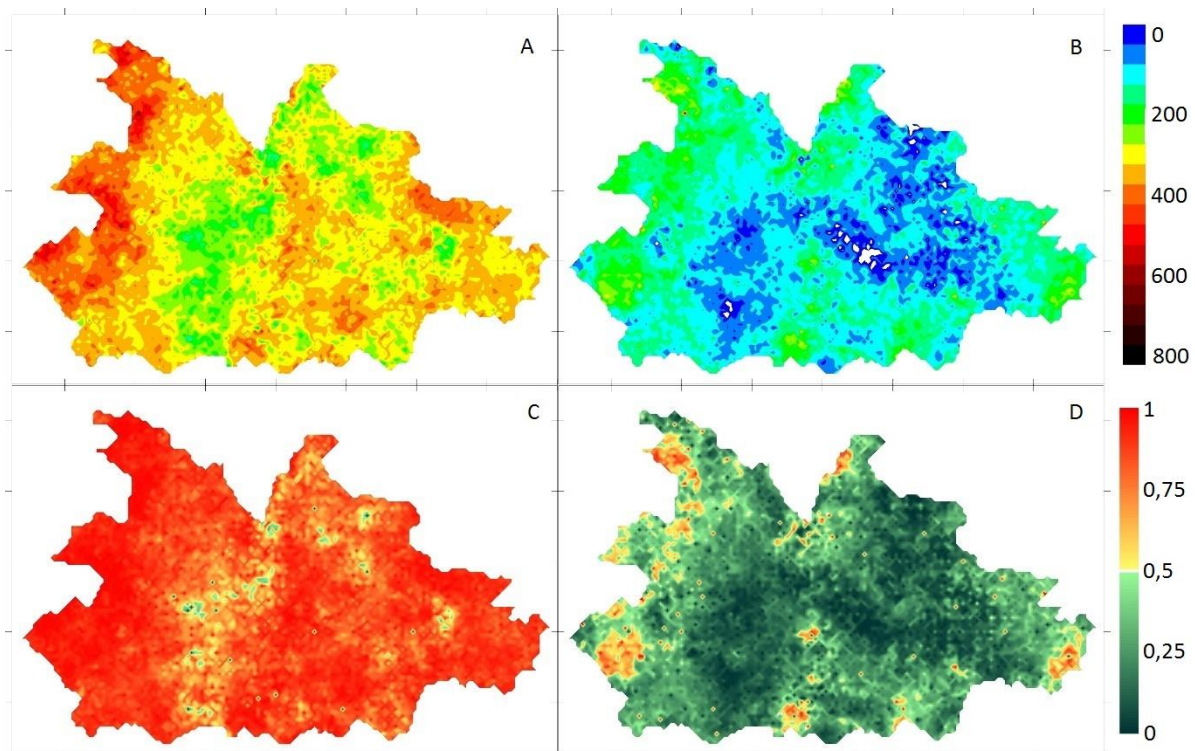


Fig. 9 Stochastic evaluation of 125 joint spatio-temporal stochastic simulation. E-type estimation of the groundwater depth in 12/10/2009 (A), in 24/03/2011 (B) and the probability of the groundwater deeper than 2 meters in the similar time instants (C and D respectively)

available water resources on the site alongside the space-time uncertainty of the estimations. The methodology can be well supplied to determine spatial and temporal patterns of the groundwater resources.

After the 1970s in Hungary, a serious groundwater discharge had occurred on the Sand ridge, between the river Danube and the Tisza. In some particular sites this depletion seems irreversible. The main causes are attributed to water extraction and the increasing area of forests. Nevertheless, our study shows that in a decisive part of the area, besides groundwater depletion the meteorological (and climatic) changes can be identified. Specifically, in the whole Danube-Tisza interflow, whose area is about twice as large as the area of the western part of our study site (to the west of the Tisza river), cca. 2 km³ of potable water was exploited in the recent 40 years, while the effect of a specific drought year could exceed this volume.

Though, instead of the continuously cumulating effect of water exploitation, the resource trend pointed to the close relationship with rainfall conditions. Investigations on the other hand highlighted indirect (stealthy) effects of climate change. Due to the decrease of the arriving precipitation on the Tisza catchment, the water resources of the river were reduced. This also had a negative effect on the groundwater resources.

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