A family of random graph evolution models with moderate density

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Network theory is important both for real life applications and theoretical research. One can find several general facts on network theory in the book of Barabási [1]. An evolving network can be described by a random graph. The vertices of the graph are the nodes of the network and the edges of the graph are the connections among the nodes.

In this paper, we study a discrete time network evolution model. The evolution of the graph is based on constructions and deletions of k-cliques, where $k \geq 2$ is a fixed integer. A k-clique is a sub-graph containing k vertices and any two different vertices are connected by 1 edge. When we form a k-clique, then we draw $\binom{k}{2}$ new edges among k vertices, and we add this new clique to the list of k-cliques.

The initial graph at time $n = 0$ contains k vertices and no one edge. In the first step i.e. when the time is $n = 1$, we connect the k vertices to obtain a single k-clique. Then, in each step, we choose k vertices uniformly at random from the existing vertices. If they do not form a k-clique, then we construct a new k-clique on these vertices. In the other case, when the sub-graph consisting of the k vertices chosen is a k-clique, then that k-clique is deleted. Then a new vertex is added to the graph and two new k-cliques are created.

Using martingale theory, we prove almost sure limit theorem for the number of vertices, then we show its asymptotic normality. Then we obtain almost sure limit theorem for the degree of a fixed vertex.

Our results are extensions of the results of Móri and Backhausz [2]. We remark, that instead of uniform choice, one can use preferential attachment principle for certain sub-graphs, but then the asymptotic behaviour of the graph will be different, see e.g. [3].

[1] A.-L. Barabási, *Network science*, Cambridge University Press, Cambridge, UK, 2018.

[2] Á.Backhausz, T. F. Móri, A random graph of moderate density. *Electron. Commun. Probab.* **27**(2022), 1–12.

[3] I. Fazekas, Cs. Noszály, A. Perecsényi, The N-star network evolution model, *J. Appl. Probab.* **56**(2019), No. 2, 416–440.

Fazekas Péter

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Azzal foglalkozunk, hogy jobban megértsük a híres LLL (Lenstra, Lenstra, Lovász) és hasonló bázisredukciós algoritmusok geometriai hátterét. Ezen algoritmusok célja, hogy egy